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Towards 5D Grand Unification without SUSY Flavor Problem

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Abstract: We consider the renormalization group approach to the SUSY flavor problem in the supersymmetric SU(5) model with one extra dimension. In higher dimensional SUSY gauge theories, it has been recently shown that power corrections due to Kaluza-Klein states of gauge fields run the soft masses generated at the orbifold fixed point to flavor conserving values in the infra-red limit. In models with GUT breaking at the brane where the GUT scale can be larger than the compactification scale, we show that the addition of a bulk Higgs multiplet, which is necessary for the successful unification, is compatible with the flavor universality achieved at the compactification scale.

Keywords: Renormalization Regularization and Renormalons, Renormalization Group, Supersymmetry Breaking, GUT.

Contents

1. Introduction

Supersymmetry(SUSY) has been one of the most elegant candidates for solving the hierarchy problem in the Standard Model(SM) [1]. However, the general SUSY breaking with soft mass terms would give rise to over one hundred independent new parameters, which would lead to less predictivity. Moreover, there could appear flavor changing processes or CP violating processes, which brings us to the so called the SUSY flavor problem [2, 3]. In the general hidden sector model for SUSY breaking, the Kähler potential in the supergravity lagrangian contains counterterms of $O(1)$ coupling between visible and hidden sectors, which give rise to flavor dependent scalar masses upon SUSY breaking. This can be understood from the fact that with flavor violation coming from the Yukawa sector, the running squark mass has a logarithmic dependence on the renormalization scale [4]. To avoid the SUSY flavor problem, there have been suggestions for the SUSY breaking mechanism: gauge-mediation [5], effective SUSY [6], anomaly-mediation [7] and etc.

For recent years, there have been alternatives to SUSY in the brane models with compact extra dimension(s) to explain the hierarchy problem without supersymmetry [8, 9]. The common assumption in these approaches is that the gauge and matter fields are confined on the (3+1)-dimenisional subspace, so called 3-brane, embedded in $D > 4$ dimensions, and the higher-dimensional fundamental scale or the physical scale at the brane can be of order of TeV scale without introducing a large hierarchy. However, it is difficult to maintain the idea of Grand Unification in these brane models with a low energy scale.

On the other hand, there were other attempts in the brane models to accommodate the Grand Unification at the intermediate scale by considering the power running of gauge couplings due to the Kaluza-Klein(KK) modes of bulk gauge and/or matter fields between the GUT scale and the compactification scale [10, 11]. In this case, supersymmetry should be assumed for naturalness of introducing the intermediate GUT scale. Therefore, we have to assume the higher dimensional SUSY for the Grand Unification in the brane models.

This idea of power running has been extended to the case with soft mass parameters in the higher dimensional GUT theories with broken SUSY [12, 13]. It has been shown that for the bulk GUT theory with only gauge fields compactified on an orbifold, the soft masses at the orbifold fixed points powerly run to flavor conserving values in the infra-red limit going down to the GUT scale, due to the KK contributions of gauge fields to the renormalization group equations (RGEs). In higher than five dimensions, the consistent models in 6D are restricted to some of SO($2N$) models and exceptional groups due to the anomaly cancellation condition [13]. Moreover, in higher than six dimensions, the bulk supersymmetry becomes of $\mathcal{N}=4$ in 4D and then the $\mathcal{N}=4$ super Yang-Mills multiplet gives rise to a zero power-like beta function of the gauge coupling.

In this paper, we reconsider the renormalization group approach to the SUSY flavor problem in 5D models on S^1/Z_2 where the GUT scale can be larger than the compactification scale. Then, we show that the non-universal KK contributions to the soft masses between the GUT scale and the compactification scale are flavor diagonal. We can also consider the matter fields in the bulk as far as the beta function coefficient of the zero-mode gauge coupling is power-like and negative. In fact, introduction of bulk matter fields is necessary for the more successful gauge unification. Particularly, the case with the down type Higgs in the bulk and the up type Higgs on the brane was considered before for explaining the top-bottom mass hierarchy [14]. In this case, however, there appear additional power corrections to the RGEs due to the KK modes of the bulk Higgs which are flavor dependent. Nonetheless, we show that the additionally generated flavor violation is negligible thanks to the small down type Yukawa coupling. On the other hand, we get a suppression of new CP phases coming from the soft terms given at $M_{\rm GUT}$ up to 10^{-2} in either case without or with one bulk Higgs.

The $\mathcal{N}=2$ SUSY in 5D is broken to $\mathcal{N}=1$ after orbifold compactification. Then, we regard the final $\mathcal{N}=1$ SUSY and the GUT symmetry as being broken at the branes. In this case, the mass spectrum of the KK modes of bulk gauge fields are modified due to the brane mass terms [15, 16, 17]. In particular, in the presence of the GUT breaking larger than the compactification scale, the lowest X, Y gauge bosons obtain masses of order the compactification scale. When the GUT scale is of order the conventional 4D SUSY GUT scale, the lower X, Y gauge boson masses could give rise to the rapid proton decay. However, since the wave functions of bulk X, Y gauge bosons are suppressed at the brane with GUT breaking, the effective suppression scale of dimension-six proton decay operator becomes of order the GUT scale.

Our paper is organized as follows. In the next section, we present our model setup with localized SUSY and GUT breakings. Then, in section 3, we give the mass spectrums and wave functions of KK modes of the whole bulk gauge multiplet and those of bulk Higgs multiplets in the presence of the localized symmetry breakings. In section 4, we consider the running of gauge couplings and comment on the proton decay problem in the models without or with bulk Higgs fields. In section 5, we go on to discuss the mass correction of the GUT scalar on the brane due to the modified KK modes. In section 6, we also present the renormalization group equations for the soft mass parameters above and below the GUT scale and then show that the soft masses converge more into flavor conserving fixed points in going down to the compactification scale. In section 7, we present the RGEs for the case with bulk Higgs fields and discuss the SUSY flavor problem in the model with one bulk Higgs field in view of the new sources of flavor violation in RGEs. Finally, we will come to an end with the conclusion.

2. Setup

Let us consider a 5D SUSY SU(5) GUT where one extra dimension is compactified on S^1/Z_2 with the radius R. It gives two fixed points at $y=0$ and $y=\pi R$. We assume that in the bulk there exists only the $\mathcal{N}=2$ SU(5) gauge fields, which contain an $\mathcal{N}=1$ vector multiplet (A_μ, λ_1) and an $\mathcal{N}=1$ chiral multiplet (Φ, λ_2). All the other matter and Higgs multiplets are assumed to live at the fixed point $y=0$. The orbifold boundary condition breaks the bulk $\mathcal{N}=2$ supersymmetry down to the $\mathcal{N}=1$ supersymmetry, but does not break the bulk gauge symmetry.

Including the $\mathcal{N}=2$[1] SUSY SU(5) gauge fields in the bulk and the $\mathcal{N}=1$ SUSY SM matter fields($3(\bar{\mathbf{5}}+\mathbf{10})$) and Higgs fields[2] ($H=\mathbf{5_H}$, $\bar{H}=\bar{\mathbf{5}}_\mathbf{H}$) on the brane at $y=0$, the 5D action is given by

$$
\begin{aligned}
S_0 &= \int d^4x dy (\mathcal{L}_5 + \delta(y)\mathcal{L}_4)\,, && (2.1)\\
\mathcal{L}_5 &= \mathrm{Tr}\left[-\frac{1}{2}(F_{MN})^2 + (D_M\Phi)^2 + \left(\bar{\lambda}_i i\gamma^M D_M \lambda^i\right) + \left(\vec{X}\right)^2 - \bar{\lambda}_i\left[\Phi, \lambda^i\right]\right], && (2.2)\\
\mathcal{L}_4 &= \int d^2\theta d^2\bar{\theta}\, \times \\
&\quad \times \left(\Psi_i(\mathbf{10})^\dagger e^{2g_5 V}\Psi_i(\mathbf{10}) + \Psi_i(\bar{\mathbf{5}})^\dagger e^{-2g_5 V}\Psi_i(\bar{\mathbf{5}}) + H^\dagger e^{2g_5 V} H + \bar{H}^\dagger e^{-2g_5 V}\bar{H}\right) + \\
&\quad + \left[\int d^2\theta \left(\frac{Y_U^{ij}}{4} H\Psi_i(\mathbf{10})\Psi_j(\mathbf{10}) + \sqrt{2}Y_D^{ij}\bar{H}\Psi_i(\mathbf{10})\Psi_j(\bar{\mathbf{5}})\right) + \text{h.c.}\right] && (2.3)
\end{aligned}
$$

where we omitted the contraction of group indices, and i,j run over 3 families of the Standard Model and g_5 is the 5D SU(5) gauge coupling with mass dimension $-\frac{1}{2}$. Note that in our convention, the covariant derivatives are defined as $D_M\Phi = \partial_M - ig_5[A_M, \Phi]$(similarly for λ^i) and $D_\mu\Psi_i = (\partial_\mu - ig_5 A_\mu)\Psi_i$. The auxiliary field D belonging to the $\mathcal{N}=1$ vetor multiplet on the boundary is given by $D = X^3 - \partial_y\Phi$.

[1]Let us borrow the notations of Mirabelli and Peskin's [18]

[2]We will also consider the case with Higgs field(s) propagating in the bulk. In this case, each Higgs multiplet comes as the zero mode of a bulk hypermultiplet.

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Now let us introduce a Higgs field $\mathbf{24} = \Sigma$ at $y = 0$ to break the GUT symmetry into the SM one

$$\mathcal{L}_\Sigma = \delta(y)\left[\int d^2\theta d^2\bar{\theta}\, 2\mathrm{Tr}\left(\Sigma^\dagger e^{2g_5 V}\Sigma e^{-2g_5 V}\right) + \left(\int d^2\theta\, W(\Sigma) + \mathrm{h.c.}\right)\right] \tag{2.4}$$

with

$$W(\Sigma) = \frac{1}{2}\mu_\Sigma \mathrm{Tr}\Sigma^2 + \frac{1}{3}Y_\lambda \mathrm{Tr}\Sigma^3 + Y_\Sigma H\Sigma\bar{H} + \mu_H H\bar{H} \tag{2.5}$$

where Y_λ and Y_Σ are dimensionless parameters, and μ_Σ and μ_H are dimensionful parameters.[3] Then, the SU(5) GUT symmetry is broken by a vacuum expectation value of the Σ field

$$\langle\Sigma\rangle = V\begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \tag{2.6}$$

with $V = \mu_\Sigma / Y_\lambda$, which gives brane masses to X, Y gauge bosons

$$M_X = M_Y = \left(5\sqrt{2}g_5 V\right)^2 \equiv M_V\,. \tag{2.7}$$

For $\mu_H = 3Y_\lambda V$, the SM Higgs doublets are massless while the color-triplet Higgs fields are superheavy as

$$M_{H_C} = M_{H_{\bar{C}}} = 5Y_\lambda V \tag{2.8}$$

which will be denoted by κ later in the case with bulk Higgs multiplets. In the case with one(two) bulk Higgs multiplet(s), κ has a mass dimension of $\frac{1}{2}(0)$.

The SM group components of the adjoint Higgs multiplet also obtain masses of the GUT scale after the GUT breaking.[4] On the other hand, for the broken components of the adjoint Higgs, say the X, Y directions, Goldstone bosons coming from $\Sigma^{(\bar{3},2)}$ and $\Sigma^{(3,2)}$ are eaten up by the X, Y gauge bosons while their physical components and the X, Y gauginos with adjoint higgsino components get masses of M_V to make up an $\mathcal{N} = 1$ massive vector multiplets together with the zero modes of X, Y gauge bosons. Since all the KK modes of bulk gauge fields are coupled to the adjoint Higgs fields on the brane, masses of KK $\mathcal{N} = 1$ massive X, Y gauge multiplets are also strongly affected by the GUT breaking effect on the brane.

Next let us add soft SUSY breaking terms for the brane fields at $y = 0$ as in the 4D SUSY SU(5) case except the gaugino mass terms. We assume that the gauge invariant soft masses for gauginos generically appear at both fixed points with arbitrarily different values as

$$\mathcal{L}_{soft} \supset -\delta(y)\left(\frac{1}{2}\varepsilon_0\lambda_1\lambda_1 + \mathrm{h.c.}\right) - \delta(y - \pi R)\left(\frac{1}{2}\varepsilon_\pi\lambda_1\lambda_1 + \mathrm{h.c.}\right) \tag{2.9}$$

[3]For the case with one(two) bulk Higgs multiplet(s), the dimensions of couplings become $[Y_\Sigma] = -\frac{1}{2}(-1)$ and $[\mu_H] = \frac{1}{2}(0)$.

[4]$\Sigma^{(8,1)}$ and $\Sigma^{(1,3)}$ have $M_\Sigma \equiv 5\mu_\Sigma/2$ and $\Sigma^{(1,1)}$ has mass $0.2M_\Sigma$.

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where $\varepsilon_{0,\pi}$ are dimensionless quantities of $\mathcal{O}(10^{-12})$ to represent the weak scale supersymmetry breaking. Note that we do not include soft terms for λ_2 at the fixed points because those will not affect the equations of motion for gauginos due to the odd parity of λ_2 on S^1/Z_2. λ_2 only participates in modifying the mass spectrum via mixing with λ_1 in the bulk. For softness as will be shown later, we assume that ε_0 is zero. In the case with bulk Higgs fields, we introduce soft mass terms for bulk Higgs scalars only at $y = \pi R$ for the same reason.

3. Mass spectrum with brane-induced SUSY and gauge symmetry breakings

Firstly, let us give a brief review on the KK mass spectrum of the X, Y gauge bosons in the presence of their localized gauge symmetry breaking [16, 17]. After the GUT breaking by the VEV of Σ, the X, Y gauge bosons acquire brane-localized masses,

$$\mathcal{L} \supset \delta(y)\frac{1}{2}M_V A^{\hat{a}}_\mu(x,y)A^{\hat{a}\mu}(x,y) \tag{3.1}$$

where $\hat{a}$ runs over broken generators.

Then, under the KK reduction, the X, Y gauge bosons for $-\pi R < y < \pi R$ can be written as

$$A^{\hat{a}}_\mu(x,y) = \frac{1}{\sqrt{\pi R}}\sum_n N_n A^{\hat{a}(n)}_\mu(x)\cos(M^A_n|y| - \theta^A_n) \tag{3.2}$$

satisfying the equation of motion

$$-\partial_y^2 A^{\hat{a}}_\mu(x,y) + \delta(y)M_V A^{\hat{a}}_\mu(x,y) = (M^A_n)^2 A^{\hat{a}}_\mu(x,y)\,. \tag{3.3}$$

Consequently, with $\theta^A_n = M^A_n\pi R$, the mass spectrum is determined by the boundary condition at $y = 0$

$$\tan(M^A_n\pi R) = \frac{M_V}{2M^A_n}. \tag{3.4}$$

In the limit $M_V \gg M^A_n$, we can obtain the approximate masses as

$$M^A_n \simeq \left(n+\frac{1}{2}\right)M_c\left\{1 - \frac{2M_c}{\pi M_V} + \left(\frac{2M_c}{\pi M_V}\right)^2\right\} \tag{3.5}$$

where $M_c = 1/R$ and $n = 0, 1, 2, \ldots$. Therefore, even with the large gauge boson masses on the brane, the lowest KK modes of X, Y gauge bosons get masses of order the compactification scale. This fact is related to the modified wave function of X, Y gauge bosons, which are repelled from the brane at $y = 0$ due to the brane mass term. Thus, it gives rise to the suppression of effective gauge coupling at the brane, so that the X, Y gauge boson mass of order M_c much lower than the GUT scale can be consistent with the proton stability [16]. When we integrate the 5D action for the gauge bosons over the extra dimension with eq. (3.2), the normalization constant N_n is determined by

$$N_n = \left(1 + \frac{M_c M_V}{2\pi(M^A_n)^2}\cos^2\theta^A_n\right)^{-1/2}. \tag{3.6}$$

Since the SM gauge bosons do not have brane masses, their KK mass spectrum is just given as $M_n = nM_c$ with $n = 0, 1, 2, \ldots$ and the normalization becomes $N_n = 1/\sqrt{2^{\delta_{n,0}}}$.

Let us take into account the mass spectrum of X, Y gauginos in the presence of their Dirac mass terms with X, Y components of brane higgsinos at $y = 0$ and their soft mass terms at both fixed points. The mass eigenstates of X, Y components of adjoint fermions will be written as a linear combination of the KK modes of $\lambda_{1,2}$ and $\tilde{\Sigma}$ due to bulk and brane mixings. To obtain the mass spectrum and the mass eigenstates of gauginos, we have to solve the following field equations

$$i\bar{\sigma}^\mu \partial_\mu \tilde{\Sigma}^b - \sqrt{M_V}\overline{\lambda_1^a}|_{y=0} = 0 \tag{3.7}$$

$$i\bar{\sigma}^\mu \partial_\mu \lambda_2^a + \partial_y \overline{\lambda_1^a} = 0 \tag{3.8}$$

$$i\bar{\sigma}^\mu \partial_\mu \lambda_1^a - \partial_y \overline{\lambda_2^a} - \left(\sqrt{M_V}\overline{\tilde{\Sigma}^b} + \varepsilon_0 \overline{\lambda_1^a}\right)\delta(y) - \varepsilon_\pi \overline{\lambda_1^a}\delta(y - \pi R) = 0 \tag{3.9}$$

where a, b run over inequivalent pairs of two nearest indices of broken generators.

Let us take the form of the gaugino solutions to be consistent with the orbifold symmetry for $-\pi R < y < \pi R$,

$$\lambda_1^a(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_n K_n \eta_1^{a(n)}(x) \cos\left(M_n^X |y| - \theta_n^X\right) \tag{3.10}$$

$$\lambda_2^a(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_n K_n \eta_2^{a(n)}(x) \epsilon(y) \sin\left(M_n^X |y| - \theta_n^X\right) \tag{3.11}$$

where $\epsilon(y)$ is a sign function. If we also assume

$$\eta_1^{a(n)}(x) = \eta_2^{a(n)}(x) \equiv \lambda^{a(n)}(x)\,, \tag{3.12}$$

$$\tilde{\Sigma}^a(x) = \frac{1}{\sqrt{2\pi R}} \sum_n K_n \frac{\sqrt{M_V}}{M_n^X} \cos\theta_n^X \lambda^{a(n)}(x)\,, \tag{3.13}$$

then we obtain the solutions for θ_n^X and M_n^X from the boundary conditions at the branes

$$\tan\theta_n^X = \frac{M_V}{2M_n^X} + \frac{\varepsilon_0}{2}\,, \tag{3.14}$$

$$\tan\left(M_n^X \pi R - \theta_n^X\right) = \frac{\varepsilon_\pi}{2}\,. \tag{3.15}$$

Therefore, the eigenvalue equation for the X, Y gaugino masses are

$$\tan\left(M_n^X \pi R - \arctan\left(\frac{\varepsilon_\pi}{2}\right)\right) = \frac{M_V}{2M_n^X} + \frac{\varepsilon_0}{2}\,. \tag{3.16}$$

Thus, we obtain the approximate mass spectrum of X, Y gauginos as

$$M_n^X \simeq M_n^{X(0)}\left\{1 - \frac{2M_c}{\pi M_V}\left(1 + \frac{\varepsilon_0 M_n^{X(0)}}{M_V}\right)^{-1} + \left(\frac{2M_c}{\pi M_V}\right)^2\left(1 + \frac{\varepsilon_0 M_n^{X(0)}}{M_V}\right)^{-3}\right\} \tag{3.17}$$

where

$$M_n^{X(0)} = M_c\left[\left(n + \frac{1}{2}\right) + \frac{1}{\pi}\arctan\left(\frac{\varepsilon_\pi}{2}\right)\right] \tag{3.18}$$

with integer n. We can also write the 4D effective action for the adjoint fermions in terms of the mass eigenstates $\lambda^{a(n)}(x)$ to obtain the normalization constant as

$$K_n = \left(1 + \frac{M_c M_V}{2\pi (M_n^X)^2}\cos^2\theta_n^X\right)^{-1/2}. \tag{3.19}$$

The mass spectrum of the SM gauginos can be obtained by nullifying the GUT breaking on the brane in eqs. (3.14), (3.16) and (3.19) as

$$\begin{aligned} \tan\theta_n^{SM} &= \frac{\varepsilon_0}{2}, & (3.20)\\ \tan(M_n^{SM}\pi R - \arctan(\frac{\varepsilon_\pi}{2})) &= \frac{\varepsilon_0}{2}, & (3.21)\\ K_n^{SM} &= 1. & (3.22) \end{aligned}$$

Then, we obtain the SM gaugino mass spectrum as

$$\begin{aligned} M_n^{SM} &= nM_c + \frac{M_c}{\pi}\arctan(\frac{\varepsilon_0}{2}) + \frac{M_c}{\pi}\arctan(\frac{\varepsilon_\pi}{2}) \\ &= nM_c + \frac{M_c}{\pi}\arctan\left[\frac{\varepsilon_0+\varepsilon_\pi}{2-\frac{\varepsilon_0\varepsilon_\pi}{2}}\right] \end{aligned} \tag{3.23}$$

where n is an integer.

Let us remark on the restoration of supersymmetry in our setup. When we take the SM gaugino masses with opposite signs at the branes, $\varepsilon_0 = -\varepsilon_\pi$, there is no modification of the KK masses, which implies that supersymmetry can be restored for the SM gauginos even if the KK modes are mixed to make up the mass eigenstates. However, this is not the case for the X, Y gauginos. Even with $\varepsilon_0 = -\varepsilon_\pi$, there is generically no restoration of supersymmetry for the X, Y gauginos since there is the mixing between gauginos and Higgsinos at $y = 0$.

Furthermore we comment on the couplings of gauge bosons and gauginos in view of the modified wave functions. To begin with, let us take into account the case with the SM gauge fields. In the presence of nonzero gaugino masses at both fixed points, the coupling of the KK mode of SM gauginos to brane matters at $y = 0$ is proportional to $\cos\theta_n^{SM}$, which is generically different from that of the corresponding KK mode of the SM gauge bosons. Then, softness of the SM gaugino masses on the brane can be guaranteed only if $\cos\theta_n^{SM} = 1$, i.e., there is no gaugino mass at $y = 0$. Therefore, for our discussion, henceforth we assume $\varepsilon_0 = 0$. On the other hand, for each pair of X, Y gauge bosons and gauginos, we also have different couplings at $y = 0$ proportional to $N_n \cos\theta_n^A$ and $K_n \cos\theta_n^X$, respectively. Even with $\varepsilon_0 = 0$, however, couplings of X, Y gauge bosons and gauginos are not the same, so there would appear quadratic divergences to soft masses for brane fields from loop corrections. Let us postpone the detailed discussion on this issue to the section 5.

For completeness with gauge multiplet, we also add the mass spectrum for the bulk real scalar field Φ coupled to the adjoint Higgs Σ on the brane. The relevant lagrangian is

$$\mathcal{L} \supset \frac{1}{2}D^aD^a + D^a(\partial_y\Phi^a) + \frac{1}{2}(\partial_\mu\Phi^a)(\partial^\mu\Phi^a) + \delta(y)\left[(\partial_\mu \mathrm{Im}\Sigma^a)^2 + \sqrt{2M_V}D^b\mathrm{Im}\Sigma^a\right] \tag{3.24}$$

where the last term is nonzero only for (a,b) of two nearest indices of broken generators and we wrote the action in terms of the auxiliary field on the boundary, $D^a = X^{3a} - \partial_y \Phi^a$. Here we note that the real part of the X,Y components of Σ can be gauged away to give the longitudinal degree of freedom to the X,Y gauge bosons while the imaginary part of the X,Y components of Σ gets mass due to the mixing with Φ at the brane. The equations of motion for the X,Y components of scalars become

$$D^a + \partial_y \Phi^a + \sqrt{2M_V}\mathrm{Im}\Sigma^b \delta(y) = 0 \tag{3.25}$$

$$-\partial^\mu \partial_\mu \Phi^a - \partial_y D^a = 0 \tag{3.26}$$

$$-2\partial^\mu \partial_\mu \mathrm{Im}\Sigma^b + \sqrt{2M_V} D^a|_{y=0} = 0 . \tag{3.27}$$

Then, we can find the solutions for the scalars for $-\pi R < y < \pi R$ as

$$\Phi^a(x,y) = \frac{1}{\sqrt{\pi R}} \sum_n C_n \epsilon(y) \sin(M_n^\Phi |y| - \theta_n^\Phi)\varphi^{a(n)}(x) , \tag{3.28}$$

$$\mathrm{Im}\Sigma^b(x) = \frac{1}{\sqrt{\pi R}} \sum_n C_n \frac{\sqrt{M_V}}{M_n^\Phi}(\cos\theta_n^\Phi)\varphi^{a(n)}(x) , \tag{3.29}$$

$$D^a(x,y) = \frac{1}{\sqrt{\pi R}} \sum_n C_n \cos(M_n^\Phi |y| - \theta_n^\Phi) D_n^a(x) , \tag{3.30}$$

where $\theta_n^\Phi = \theta_n^A = M_n^A \pi R$ and $M_n^\Phi = M_n^A$ from the boundary condition at the brane and the normalization constant C_n is also the same as N_n for the X,Y gauge bosons. Therefore, the mass spectrum for the bulk real scalar field Φ is the same as the one for the gauge bosons as required from the 4D $\mathcal{N}=1$ massive supersymmetry.

From the action eq. (2.3), the matter scalar ϕ also couples to the auxiliary field D. Then, we can integrate out the KK modes of the auxiliary field D by using the following equation of motion

$$D_n^a + M_n^A \varphi_n^a + \frac{g_5}{\sqrt{\pi R}} N_n (\cos\theta_n^A)\phi^\dagger T^a \phi = 0 . \tag{3.31}$$

This gives the interaction term between the matter scalar ϕ and the KK modes of the bulk real scalar $\varphi^{(n)}$ and the self interaction term as

$$\int d^4x \sum_n \Big[-\frac{g_5}{\sqrt{\pi R}} N_n M_n^A (\cos\theta_n^A)\phi^\dagger \varphi^{(n)}\phi - \frac{g_5^2}{2\pi R} N_n^2 (\cos\theta_n^A)^2 (\phi^\dagger T^a \phi)^2 \Big] . \tag{3.32}$$

Now let us close this section with the case that Higgs multiplets propagate in the bulk. The KK mass spectrum of colored Higgs triplets is also modified due to the brane mass term after the GUT breaking, which can be written in terms of $4D$ $\mathcal{N}=1$ superfields as $(\kappa H_C H_{\bar{C}} + \text{h.c.})\delta(y)$ where $\kappa \simeq 5Y_\Sigma V$ from eq. (2.8) in case of $\mu_H = 3Y_\lambda V$ being not exact. On the other hand, the KK mass spectrum of the bulk Higgs doublets is also modified for the nonvanishing brane mass term, $(\varepsilon_H H_u H_d + \text{h.c.})\delta(y)$ where $\varepsilon_H = \mu_H - 3Y_\lambda V$.

The case with two Higgs multiplets($H = H_C + H_u, \bar{H} = H_{\bar{C}} + H_d$) in the bulk, for which κ and ϵ_H are dimensionless, was dealt with in ref. [16]. In the presence of the brane mass term for the bulk colored higgsinos, it has been shown that the zero modes of two bulk higgsinos get a Dirac mass. From the eigenvalue equation, $\tan^2(M_n^{H_C}\pi R) = \kappa^2/4$ given in [16], the KK mass spectrum of colored higgsinos is given by

$$M_n^{H_C} = nM_c + \frac{M_c}{\pi}\arctan\left(\frac{\kappa}{2}\right) \tag{3.33}$$

where n is an integer. Likewise, the KK mass spectrum of the higgsino doublets is the one with κ replaced by ε_H in the above equation. Then, the lowest higgsino doublet mass is approximately given by $\varepsilon_H M_c/(2\pi)$, which corresponds to the μ parameter of order TeV in the MSSM after a fine-tuning between GUT parameters.

Here we also consider the case with one Higgs(H) on the brane and the other Higgs$(\bar{H})$ in the bulk, in which case κ and ε_H have a mass dimension $\frac{1}{2}$. Without a brane mass term for higgsinos, $\bar{H}$ comes from the zero mode of a bulk hypermultiplet composed of $(\bar{H} = H_{\bar{C}} + H_d, \bar{H}^c = H^c_{\bar{C}} + H^c_d)$. When we take into account the brane mass term for the colored higgsinos at $y = 0$, which mixes brane and bulk colored higgsinos, the equations of motion for the brane and bulk colored higgsinos are given by

$$i\bar{\sigma}^\mu\partial_\mu H_C - \kappa\overline{H}_{\bar{C}}|_{y=0} = 0\,, \tag{3.34}$$

$$i\bar{\sigma}^\mu\partial_\mu H_{\bar{C}} - \partial_y\overline{H}^c_{\bar{C}} - \kappa\overline{H}_C\delta(y) = 0\,, \tag{3.35}$$

$$i\bar{\sigma}^\mu\partial_\mu H^c_{\bar{C}} + \partial_y\overline{H}_{\bar{C}} = 0\,. \tag{3.36}$$

Following the similar procedure as for the gauge multiplet, we find the solutions of the colored higgsinos for $-\pi R < y < \pi R$ as

$$H_{\bar{C}}(x,y) = \frac{1}{\sqrt{\pi R}}\sum_n N_n^H h_1^{(n)}(x)\cos\left(M_n^{H_C}(|y| - \pi R)\right), \tag{3.37}$$

$$H^c_{\bar{C}}(x,y) = \frac{1}{\sqrt{\pi R}}\sum_n N_n^H h_2^{(n)}(x)\epsilon(y)\sin\left(M_n^{H_C}(|y| - \pi R)\right), \tag{3.38}$$

$$H_C(x) = \frac{1}{\sqrt{\pi R}}\sum_n N_n^H \frac{\kappa}{M_n^{H_C}}\cos\left(M_n^{H_C}\pi R\right) h_2^{(n)}(x)\,, \tag{3.39}$$

where $i\bar{\sigma}^\mu\partial_\mu h_1^{(n)} = M_n^{H_C}\overline{h}_2^{(n)}$, $i\bar{\sigma}^\mu\partial_\mu h_2^{(n)} = M_n^{H_C}\overline{h}_1^{(n)}$, and the normalization constant N_n^H is given by

$$N_n^H = \left(1 + \frac{M_c\kappa^2}{2\pi(M_n^{H_C})^2}\cos^2\left(M_n^{H_C}\pi R\right)\right)^{-1/2}. \tag{3.40}$$

Accordingly we also obtain the eigenvalue equation as

$$\tan(M_n^{H_C}\pi R) = \frac{\kappa^2}{2M_n^{H_C}}\,. \tag{3.41}$$

Here we note that a pair of Weyl spinors, $h_1^{(n)}$ and $h_2^{(n)}$, make up a Dirac mass at each KK level. The brane higgsino participates in the mixing between KK modes to make the zero mode of the bulk higgsino massive. Therefore, for $\kappa^2 \gg M_n^{H_C}$, the approximate mass spectrum for the colored higgsinos is given by

$$M_n^{H_C} \simeq \left(n + \frac{1}{2}\right)M_c\left\{1 - \frac{2M_c}{\pi\kappa^2} + \left(\frac{2M_c}{\pi\kappa^2}\right)^2\right\} \tag{3.42}$$

where $n = 0, 1, 2, \ldots$. This result reminds us of the case with gauge multiplet without SUSY breaking when we identify κ^2 as M_V. Likewise, the eigenvalue equation for higgsino doublets is given by the one with κ replaced by ε_H in eq. (3.41). Then, for $\varepsilon_H^2 \ll M_n^H$, the mass spectrum of higgsino doublets become $M_0^H \simeq \varepsilon_H \sqrt{M_c/(2\pi)}$, and $M_n^H \simeq nM_c + \varepsilon_H^2/(2n\pi)$ for $n = 1, 2, 3, \ldots$. We also need a fine-tuning between the GUT parameters to obtain the zero mode higgsino mass around TeV.

The bulk Higgs scalars have the same modification due to the brane mass term at $y = 0$ as that of bulk higgsinos due to the remaining $\mathcal{N} = 1$ supersymmetry. Then, only taking into account the allowed gauge symmetry, we can put the SUSY breaking mass terms for the bulk Higgs scalar at both fixed points, which then make a shift of the KK spectrum of the bulk Higgs scalar. However, the SUSY breaking term at $y = 0$ gives rise to different couplings of bulk Higgs and higgsino to a brane scalar as in the case with the bulk gauge multiplet, so that the quadratic divergences are not cancelled between fermionic and bosonic loops at the brane. Therefore, we have to assume that the soft mass term for a bulk Higgs scalar appears only at $y = \pi R$.

4. Gauge coupling unification and proton decay

We have shown that the mass spectrum of the bulk gauge multiplet is modified in the existence of the mass terms coming from the supersymmetry and gauge symmetry breakings on the brane. In particular, even with the large scale masses due to the GUT breaking at the brane, the $\mathcal{N} = 1$ massive X, Y gauge multiplet get masses of order the compactification scale. The brane soft mass term for the bulk gauginos just gives rise to an overall small shift for the KK spectrum of gauginos as in figures 1 and 2.

In this section, in our model that the compactification scale M_c is smaller than the GUT scale $M_{\rm GUT}$, we consider the running of gauge couplings due to the KK modes of the bulk gauge multiplet up to the GUT scale. In this case, however, we must guarantee the successful unification which is spoiled due to the additional non-universal log running, as well as the proton longevity which is challenged due to the somewhat smaller X, Y gauge boson masses.

When we take into account the KK contributions above the compactification scale ($M_c \equiv 1/R$) ignoring the SUSY breaking effects on the KK spectrum, the running of the zero-mode gauge couplings is given by

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_{\rm GUT}) + \frac{1}{2\pi} b'_i \ln\left(\frac{M_{\rm GUT}}{\mu}\right) + \frac{1}{2\pi} b_i^{SM} \sum_{0<nM_c<M_{\rm GUT}} \ln\left(\frac{M_{\rm GUT}}{nM_c}\right) + \\ + \frac{1}{2\pi} b_i^X \sum_{M_n<M_{\rm GUT}} \ln\left(\frac{M_{\rm GUT}}{M_n}\right) \tag{4.1}$$

where M_n are the KK masses of the $\mathcal{N} = 1$ massive X, Y gauge multiplet, and $b'_i = (33/5, 1, -3)$, $b_i^{SM} = (0, -4, -6)$, $b_i^X = (-10, -6, -4)$ are the beta function coefficients for the zero mode MSSM fields, the $\mathcal{N} = 1$ massive MSSM gauge multiplet and the $\mathcal{N} = 1$ massive X,Y gauge multiplet, respectively. Then, with $N = M_{\rm GUT}/M_c$, using the Stirling's

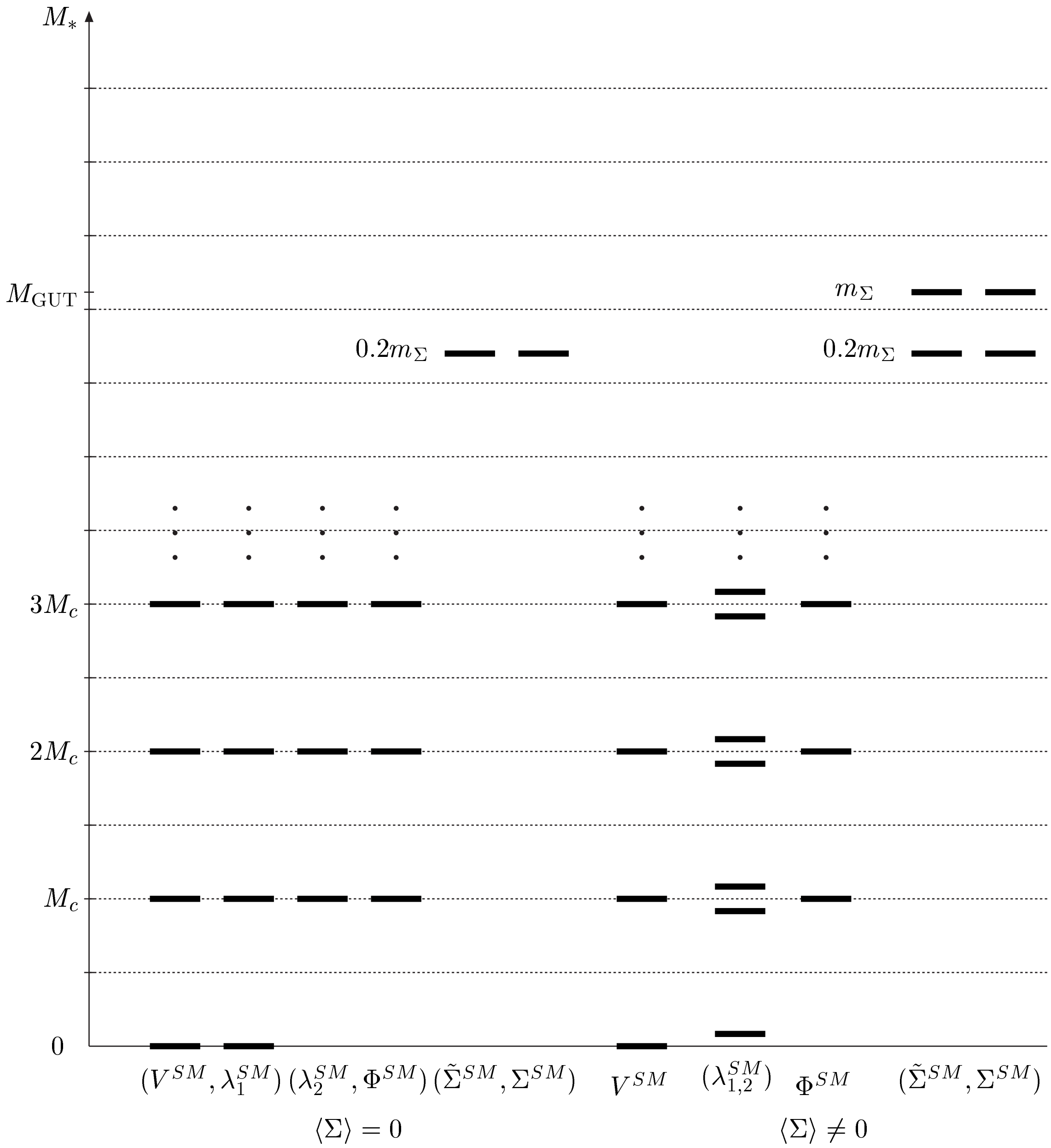

Figure 1: Mass spectrum of the SM group part before and after the GUT/SUSY breaking on the brane. After the GUT breaking, the $(8,1)$,$(1,3)$ Higgs and Higgsino has the mass m_Σ while the $(1,1)$ Higgs and Higgsino has the mass $0.2m_\Sigma$.

formular, we can get the sum of the KK modes as

$$\sum_{0<nM_c<M_{\rm GUT}} \ln\left(\frac{M_{\rm GUT}}{nM_c}\right) = \sum_{n=1}^{N-1} \ln\left(\frac{N}{n}\right) \simeq N - \frac{1}{2}\ln(2\pi N)\,, \tag{4.2}$$

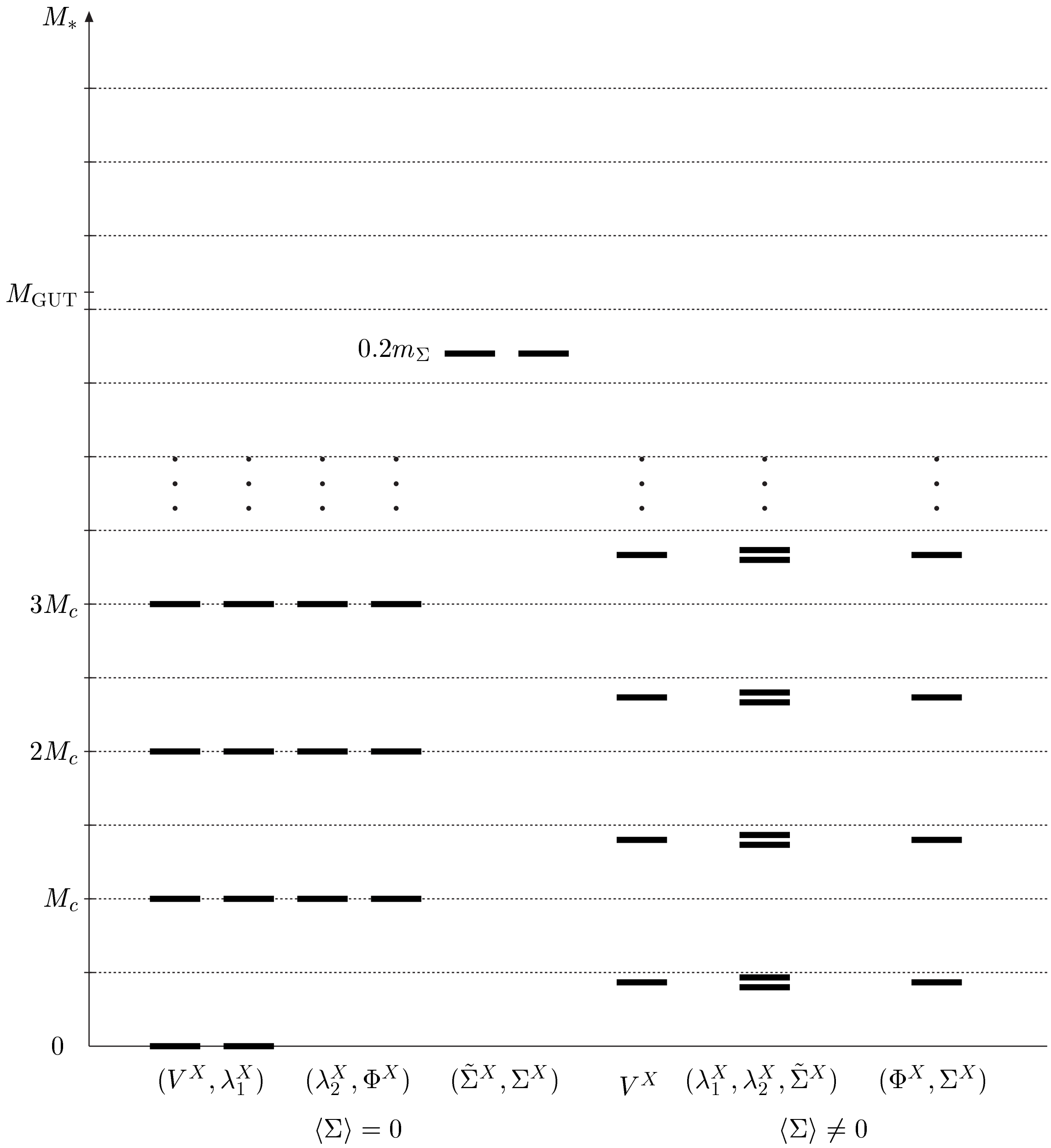

Figure 2: Mass spectrum of the X,Y broken group part before and after the GUT/SUSY breaking on the brane.

$$\sum_{M_n < M_{\mathrm{GUT}}} \ln\left(\frac{M_{\mathrm{GUT}}}{M_n}\right) = \sum_{n=1}^{N} \ln\left[\frac{2N}{(2n-1)(1-\zeta/N+(\zeta/N)^2)}\right]$$
$$\simeq N - \frac{1}{2}\ln 2 + \zeta + \mathcal{O}\left(\frac{\zeta^2}{N}\right) \qquad (4.3)$$

where

$$\zeta = \frac{2M_{\mathrm{GUT}}}{\pi M_V}\,. \qquad (4.4)$$

Therefore, the resulting running equations of low energy gauge couplings are given by

$$\begin{aligned}\alpha_i^{-1}(\mu) &= \alpha_i^{-1}(M_{\rm GUT}) + \frac{1}{2\pi}b'_i \ln\left(\frac{M_{\rm GUT}}{\mu}\right) + \frac{1}{2\pi}bN - \\ &\quad -\frac{1}{4\pi}b_i^{SM}\ln(2\pi N) - \frac{1}{4\pi}b_i^X \ln 2 + \frac{1}{2\pi}b_i^X\zeta \end{aligned}\tag{4.5}$$

where $b \equiv b_i^{SM} + b_i^X = -10$. The power running proportional to N is universal for all gauge couplings and cannot contribute in narrowing down the separations of the gauge couplings, but it leads the gauge couplings to very small ones at the unification scale since $b < 0$. That is, the zero-mode gauge coupling at the unification scale becomes $\alpha(M_{\rm GUT}) \simeq -2\pi/(bN) \sim 6.3\times 10^{-3}$ for $N = 10^2$. In our paper, we assume that the maximum number of KK modes(N) is of $\mathcal{O}(100)$ for the validity of perturbative calculations. On the other hand, the log terms still contribute in narrowing down the separations of gauge couplings toward a unified gauge coupling constant at $M_{\rm GUT}$.

A linear combination of the above running equations gives a theoretic value of α_3 as

$$\alpha_3^{-1} = \frac{12}{7}\alpha_2^{-1} - \frac{5}{7}\alpha_1^{-1} + \frac{\tilde b}{2\pi}\ln(\pi N) + \frac{\tilde c}{2\pi}\zeta \tag{4.6}$$

where $\tilde b = -\frac{1}{2}(b_3^{SM} - (12/7)b_2^{SM} + (5/7)b_1^{SM}) = -3/7$ and $\tilde c = b_3^X - (12/7)b_2^X + (5/7)b_1^X = -6/7$. Then, we obtain the KK correction to the value of α_s with $N = 10^2$ in the 4D minimal SUSY GUT as

$$\delta_{KK}\alpha_s \simeq -\frac{1}{2\pi}\alpha_s^2(M_Z)(\tilde b\ln(\pi N) + \tilde c\zeta) \simeq 0.0066 \tag{4.7}$$

which gives the strong coupling at M_Z in our 5D model as $\alpha_s^{KK} = \alpha_s^{SGUT,0} + \delta_{KK}\alpha_s$ where $\alpha_s^{SGUT,0}$ is the value from the 4D SUSY GUT without threshold corrections. Note, however, that this correction is not in the favorable direction. Thus, we find that there is a competition between the large separation of scales N and the correct value of α_s. Currently, we have the experimental data which is somewhat smaller than the MSSM prediction, $\alpha_s^{exp}(M_Z) - \alpha_s^{SGUT,0} = -0.013 \pm 0.0045$ [14]. Our KK modes positively add to the MSSM prediction as given in (4.7), and the discrepancy is little bit enlarged. However, considering the current experimental error bounds and the theoretically unknown threshold corrections at the GUT scale, we can tolerate the positive contribution of order 0.006 to the theoretical value of α_s.

If we had put some of brane fields in the bulk, we could have been in a better situation for the successful prediction for the strong coupling. For instance, let us consider the case (1) where the down-type Higgs multiplet comes from the bulk and the up-type Higgs multiplet is put on the brane [14], and the case (2) where two Higgs multiplets 5_H and $\bar 5_H$ come from the bulk. Then, as shown in the previous section, the mass spectrum of colored Higgs triplet(s) is modified due to the brane mass term. In the case (2), with κ larger than $O(1)$, the approximate KK masses for the colored Higgs triplets are $M_n^{H_C} \simeq (n+1/2)M_c - 2M_c/(\pi\kappa)$, $n \geq 0$. On the other hand, in the case (1), the approximate mass spectrum of the colored Higgs triplet for $\kappa^2 \gg M_n^H$ is $M_n^{H_C} \simeq (n+1/2)M_c(1 - \zeta'/N + (\zeta'/N)^2)$

where $\zeta' = 2M_{\rm GUT}/(\pi\kappa^2)$. But in both cases, the bulk Higgs doublet has the same mass spectrum as for the SM gauge multiplet. Therefore, there exist additional contributions coming from the KK modes of bulk Higgs multiplets on the right-hand side of eq. (4.1):

$$\frac{1}{2\pi} b_i^{H_d} \sum_{0<nM_c<M_{\rm GUT}} \ln\left(\frac{M_{\rm GUT}}{M_n}\right) + \frac{1}{2\pi} b_i^{H_C} \sum_{M_n^{H_C}<M_{\rm GUT}} \ln\left(\frac{M_{\rm GUT}}{M_n^{H_C}}\right) \tag{4.8}$$

where $b_i^{H_d}, b_i^{H_C}$ are beta function coefficients for the Higgs doublet and triplet, $b_i^{H_d} = (3/5(6/5), 1(2), 0)$, $b_i^{H_C} = (2/5(4/5), 0, 1(2))$ for one(two) bulk Higgs multiplet(s). Consequently, the additional KK contributions sum up to make the power running a bit smaller, $b \to b + b_i^{H_d} + b_i^{H_C} = -9(-8)$. On the other hand, we find that the bulk Higgs correction together with the bulk gauge contribution is now in the favorable direction toward the experimental value of the strong coupling. That is, in eq. (4.6), $\tilde{b}$ becomes $\tilde{b}+\tilde{b}^H = 3/14(6/7)$ where $\tilde{b}^H = -\frac{1}{2}(b_3^{H_d} - (12/7)b_2^{H_d} + (5/7)b_1^{H_d}) = 9/14(9/7)$ for one(two) bulk Higgs multiplet(s), and $\tilde{c}\zeta$ becomes changed to $\tilde{c}\zeta + \tilde{c}_H\zeta'$ only for the case with one bulk Higgs where $\tilde{c}_H = b_3^{H_C} - (12/7)b_2^{H_C} + (5/7)b_1^{H_C} = 9/7$. Then, we obtain the KK correction to the strong coupling as $\delta_{KK}\alpha_s \simeq -0.0015(-0.0095)$ for $N = 10^2$.

Now we can determine the unification scale in the minimal case with only gauge fields in the bulk by using running equations for α_1 and α_2 as

$$\begin{aligned} M_{\rm GUT} &= M_X (2\pi N)^{\frac{1}{2}\frac{b_1^{SM}-b_2^{SM}}{b'_1-b'_2}} \, e^{\frac{b_1^X-b_2^X}{b'_1-b'_2}(\frac{1}{2}\ln 2-\zeta)} \\ &\simeq 12 M_X \simeq 2.5\times 10^{17}{\rm GeV} \end{aligned} \tag{4.9}$$

where $M_X = M_Z\, e^{\frac{2\pi}{b'_1-b'_2}(\alpha_1^{-1}-\alpha_2^{-1})} \simeq 2\times 10^{16}$ GeV is the unification scale in the MSSM and we took $\zeta = 2/\pi$ for $M_V = M_{\rm GUT}$. Therefore, the GUT scale in our case is a bit higher than in the usual 4D SUSY GUTs. Then, with $N = M_{\rm GUT}/M_c = 10^2$, the compactification scale is given by $M_c \simeq 2.5\times 10^{15}$ GeV. On the other hand, in the case with one bulk Higgs, similarly we can get the unification scale and the compactification scale for $N = 10^2$ as $M_{\rm GUT} \simeq 1.3\times 10^{17}$ GeV and $M_c \simeq 1.3\times 10^{15}$ GeV. While introduction of one bulk Higgs gives rise to a more successful unification with the separation between the GUT scale and the compactification scale, KK modes of the bulk Higgs contribute flavor dependent power corrections to the renormalization group equations due to the Yukawa interaction. Let us tackle the more detail of this problem in the section 7.

In our model, we assume that the dimension-four operators with baryon number violation on the brane are not allowed due to R parity. Moreover, when we assume that the colored Higgsinos on the brane get masses of the GUT scale after the GUT breaking, there is no proton decay problem coming from the dimension-five operators with colored Higgsino exchanges either. Even if we had put Higgs multiplet(s) in the bulk, the effective suppression scale for the dimension-five operators can be higher enough for the proton longevity due to the suppression of the wave functions of colored Higgsinos on the brane [16].

On the other hand, one might worry about the potential rapid proton decay from the exchange of the X, Y gauge bosons of order $M_c \sim 10^{15}$ GeV, which would be expected to be the suppression scale of the dimension-six proton decay operators. However, the coupling of the lowest KK modes of the X, Y gauge bosons to fields on the brane is suppressed by a factor $\cos(M_n^A \pi R) \simeq \cos(\pi(n+1/2)(1-\zeta/N))$. Therefore, the suppression factor of the dimension-six operators becomes

$$
\begin{aligned}
2\sum_{n=0}^{\infty}\frac{\cos^2(M_n^A\pi R)}{(M_n^A)^2} &\simeq 8\sum_{n=0}^{\infty}\frac{1}{M_V^2+(2n+1)^2M_c^2} \\
&= \frac{2\pi\tanh(\pi M_V/2M_c)}{M_V M_c}
\end{aligned}
\tag{4.10}
$$

where we used the eigenvalue equation (3.4). Then, for $M_V \gg M_c$ and $N = 10^2$, the *effective mass* of the X, Y gauge bosons is $\simeq g_5 V\sqrt{M_c/2\pi} = g_4 V \simeq 0.28V$ at the GUT scale. The value of V near the 5D fundamental scale is sufficient to avoid the rapid proton decay from the dimension-six operators.

5. Mass correction of the GUT scalar and softness of brane SUSY breaking

In this section, before going into the renormalization group equations for the soft masses, let us compute the one-loop correction to the self energy of a GUT scalar ϕ located at the brane. There are five Feynman diagrams contributing to the self energy of the scalar ϕ as shown in figure 3, which are composed of four diagrams containing one bulk gauge field and one brane field and one diagram involving a self interaction of the scalar. Since the GUT symmetry is broken on the brane, we should take into account the contributions from the SM gauge fields and the X, Y gauge fields separately.

Let us start with the contribution from the SM gauge fields. As argued in the section 3, the SM gauge bosons and gauginos have the same gauge couplings to the brane scalar ϕ

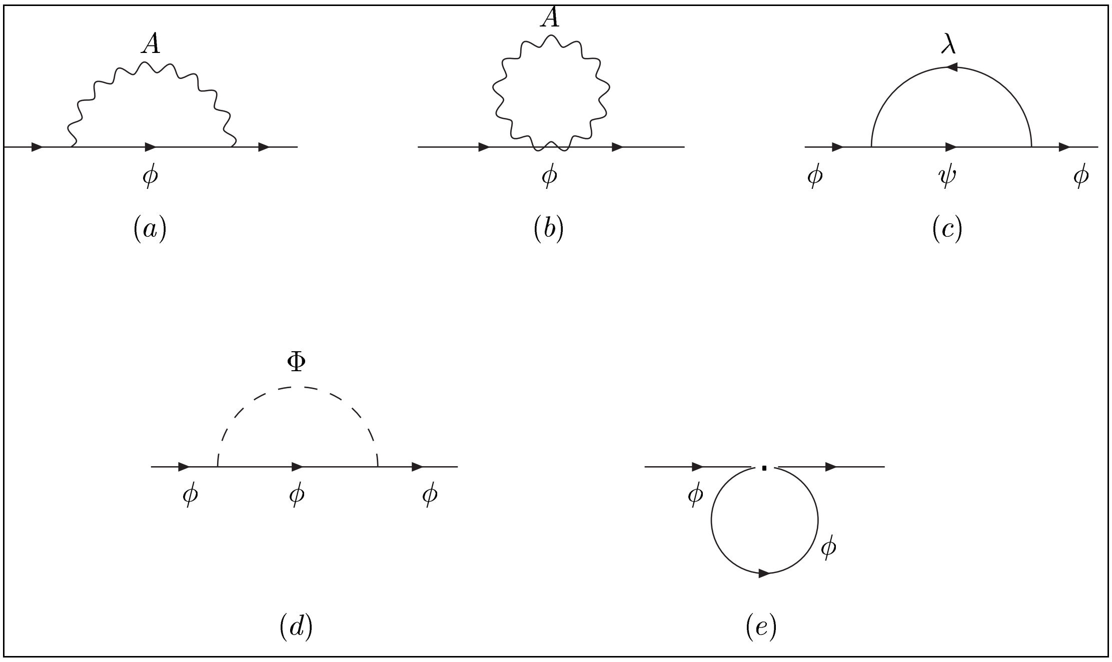

Figure 3: One-loop Feynman diagrams for the self-energy of a brane scalar ϕ.

without gaugino mass at $y=0$. When we consider the zero external momentum to see the momentum-independent mass corrections, we obtain contributions from all the five diagrams as

$$-i\,(m_\phi^2)^i_{j,SM} = g_4^2 T^a T^a \delta^i_j \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^2} \frac{1}{k^2 - M_n^2} \frac{1}{k^2 - m_\phi^2} N(k, M_n, m_\phi) \tag{5.1}$$

where i,j are generation indices, $g_4 = g_5/\sqrt{2\pi R}$, a runs over the SM generators, $M_{-n} = M_n$ and

$$\begin{aligned} N(k, M_n, m_\phi) &= -k^2 + 4(k^2 - m_\phi^2) - 4(k^2 - m_\phi^2)\frac{k^2 - M_n^2}{k^2 - (M_n^{SM})^2} + M_n^2 + (k^2 - M_n^2) \\ &= 4(k^2 - m_\phi^2)\left[\frac{M_n^2 - (M_n^{SM})^2}{k^2 - (M_n^{SM})^2}\right]. \end{aligned} \tag{5.2}$$

Therefore, we find that there is no quadratic divergence in the integrand of the 4D momentum integral, which was cancelled between the five diagrams. With the dimensional regularization ($d = 4 - \epsilon$) for the 4D momentum integral, we obtain the mass correction to the scalar ϕ as

$$\begin{aligned} (m_\phi^2)^i_{j,SM} &= \frac{4g_4^2}{16\pi^2} T^a T^a \delta^i_j \sum_{n=-\infty}^{\infty} \Bigg[\left((M_n^{SM})^2 - M_n^2\right)\left(\frac{2}{\epsilon} + \ln(4\pi) - \gamma\right) - \\ &\qquad - (M_n^{SM})^2 \ln\frac{(M_n^{SM})^2}{\mu^2} + (M_n)^2 \ln\frac{M_n^2}{\mu^2}\Bigg] \end{aligned} \tag{5.3}$$

where μ is the renormalization scale.

On the other hand, the X, Y gauge bosons and gauginos have different gauge couplings to the brane scalar ϕ even with a softness condition for the SM gauginos, $\varepsilon_0 = 0$. Therefore, quadratic divergences of the scalar mass would not be cancelled between X, Y bosonic and fermionic loop diagrams unlike for the SM gauge fields. Likewise, with the zero external momentum, contributions involving X, Y gauge fields become

$$-i\,(m_\phi^2)^i_{j,X} = g_4^2 T^{\hat a} T^{\hat a} \delta^i_j \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^2} \frac{1}{k^2 - (M_n^A)^2} \frac{1}{k^2 - m_\phi^2} N(k, M_n^A, M_n^X, m_\phi) \tag{5.4}$$

where $\hat a$ runs over broken generators, $M^A_{-n} = M^A_{n-1}$ for $n \geq 1$ and

$$\begin{aligned} N(k, M_n^A, M_n^X, m_\phi) &= (N_n^2 \cos^2\theta_n^A)\left[-k^2 + 4(k^2 - m_\phi^2) + (M_n^A)^2 + (k^2 - (M_n^A)^2)\right] - \\ &\quad -4(K_n^2 \cos^2\theta_n^X)(k^2 - m_\phi^2)\frac{k^2 - (M_n^A)^2}{k^2 - (M_n^X)^2} \\ &= 4(k^2 - m_\phi^2)\left[N_n^2 \cos^2\theta_n^A - (K_n^2\cos^2\theta_n^X)\frac{k^2 - (M_n^A)^2}{k^2 - (M_n^X)^2}\right]. \end{aligned} \tag{5.5}$$

Consequently, using the cutoff regularization for the loop integral to see the divergence structure explicitly, we find that the quadratic divergent mass correction to the scalar ϕ is non-vanishing

$$(m_\phi^2)^i_{j,X} = \frac{4g_4^2}{16\pi^2} T^{\hat a} T^{\hat a} \delta^i_j \sum_{n=-\infty}^{\infty} \Bigg[N_n^2 \cos^2\theta_n^A \left(\Lambda^2 - (M_n^A)^2 \ln \frac{\Lambda^2 + (M_n^A)^2}{(M_n^A)^2} \right) - \\ - K_n^2 \cos^2\theta_n^X \left(\Lambda^2 - (M_n^X)^2 \ln \frac{\Lambda^2 + (M_n^X)^2}{(M_n^X)^2} \right) \Bigg]. \quad (5.6)$$

Using the mass spectrum of X, Y gauge fields and considering the KK modes below the cutoff scale, we obtain the dominant piece of quadratically divergent mass correction as

$$(m_\phi^2)_X \simeq \frac{\alpha N}{\pi^3} \left(\frac{M_c}{M_V} \right)^2 \varepsilon_\pi^2 \Lambda^2 \simeq (139\text{GeV})^2 \quad (5.7)$$

where we chose the cutoff scale to be of order the GUT scale and we used $\alpha N \simeq 0.6$ at the unification scale, the number of KK modes below the cutoff scale as $N = 100$ and $M_c/M_V = 10^{-2}$. Therefore, we find that due to the partial cancellation between fermionic and bosonic contributions and the suppression of wave functions of X, Y gauge fields at the brane, the resulting quadratic divergence is sufficiently softened than the usual case without supersymmetry. Since there is no quadratic divergence in the loop correction for the gaugino mass, the gaugino mass of order the weak scale is radiatively stable, which guarantees the UV insensitivity of the one-loop scalar mass.

Then, without worrying about the UV sensitivity of the scalar mass, we can also obtain the one-loop correction from X, Y gauge fields in the dimensional regularization as

$$(m_\phi^2)^i_{j,X} = \frac{4g_4^2}{16\pi^2} T^{\hat a} T^{\hat a} \delta^i_j \sum_{n=-\infty}^{\infty} \Bigg[\left(K_n^2 \cos^2\theta_n^X (M_n^X)^2 - N_n^2 \cos^2\theta_n^A (M_n^A)^2 \right) \times \\ \times \left(\frac{2}{\epsilon} + \ln(4\pi) - \gamma \right) - K_n^2 \cos^2\theta_n^X (M_n^X)^2 \ln \frac{(M_n^X)^2}{\mu^2} + \\ + N_n^2 \cos^2\theta_n^A (M_n^A)^2 \ln \frac{(M_n^A)^2}{\mu^2} \Bigg]. \quad (5.8)$$

Consequently, adding all the gauge field contributions to the scalar mass, we get the renormalization group equation for the soft mass of the scalar ϕ as

$$\Lambda \frac{d(m_\phi^2)^i_j}{d\Lambda} = -\frac{8g_4^2}{16\pi^2} \delta^i_j \Bigg[T^a T^a \sum_{|M_n|<\Lambda} \left((M_n^{SM})^2 - M_n^2 \right) + \\ + T^{\hat a} T^{\hat a} \sum_{|M_n|<\Lambda} \left(K_n^2 \cos^2\theta_n^X (M_n^X)^2 - N_n^2 \cos^2\theta_n^A (M_n^A)^2 \right) \Bigg] \\ \simeq -\frac{8g_4^2}{16\pi^2} \left(T^a T^a + f(\Lambda) T^{\hat a} T^{\hat a} \right) \delta^i_j \left(\frac{2\Lambda}{M_c} \right) |M|^2 \quad (5.9)$$

where M is the lowest SM gaugino mass given by

$$M = \frac{1}{\pi R} \arctan \left(\frac{\varepsilon_\pi}{2} \right) \simeq \frac{\varepsilon_\pi M_c}{2\pi}, \quad (5.10)$$

and

$$f(\Lambda) = \sum_{|M_n|<\Lambda} \cos^2\theta_n^{A,X} \simeq 1 - \frac{M_V}{2\Lambda}\arctan\left(\frac{2\Lambda}{M_V}\right). \tag{5.11}$$

Here we find that the contributions coming from the KK modes of gauge fields add up to the *power running* of the scalar mass. Since $0 < f(\Lambda) < 0.45$ for $M_c < \Lambda < M_V$, contributions coming from the X, Y gauge fields are small compared to those coming from the SM gauge fields due to the suppression of their wave functions at the brane.

6. Renormalization group equations for soft masses and SUSY flavor problem

The KK modes of bulk gauge fields can also give the power running of the other soft mass parameters which are located at the brane. For convenience, we assume the boundary superpotential and the corresponding soft SUSY breaking(SSB) lagrangian at the $y = 0$ brane in the following way,

$$W(\Phi) = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu_{ij}\Phi_i\Phi_j, \tag{6.1}$$

$$-\mathcal{L}_{SSB} = \left(\frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}B^{ij}\phi_i\phi_j + \text{h.c.}\right) + \phi^{*j}(m^2)^i_j\phi_j\,. \tag{6.2}$$

Here let us remind that there exists a gaugino mass term only at the $y = \pi R$ brane.

Upon the orbifold compactification on S^1/Z_2, only the even modes of bulk fields are coupled to the brane,[5] so each KK mode gives a logarithmic contribution to two and three point functions for brane fields as in 4D [10, 19]. Therefore, logarithmic contributions to the anomalous dimension of a brane field from the KK modes below the cutoff scale sum up to the power running. For a Higgs field in the bulk, the anomalous dimension of its zero mode gets a 4D logarithmic contribution from brane fields due to Yukawa interactions at the brane while it does not have a loop contribution from the bulk gauge fields thanks to the bulk $\mathcal{N} = 2$ supersymmetry [10, 19]. For applicability to the case with extra dimension, let us recall conventional formulae for one-loop beta functions of couplings in 4D [20] as follows

$$\beta_g = \frac{g^3}{16\pi^2}\left[\sum_i l(R_i) - 3C_2(G)\right], \tag{6.3}$$

$$\beta_M = \frac{2g^2}{16\pi^2}\left[\sum_i l(R_i) - 3C_2(G)\right]M\,, \tag{6.4}$$

$$\beta_Y^{ijk} = \sum_l Y^{ijl}\gamma_l^k + (k\leftrightarrow i) + (k\leftrightarrow j)\,, \tag{6.5}$$

$$\beta_\mu^{ij} = \gamma_l^i\mu^{lj} + \gamma_l^j\mu^{il}\,, \tag{6.6}$$

$$\beta_B^{ij} = \gamma_l^i B^{lj} + \gamma_l^j B^{il} - 2\gamma_{1l}^i\mu^{lj} - 2\gamma_{1l}^j\mu^{il}\,, \tag{6.7}$$

[5] The odd modes can also couple to the brane only with their derivatives with respect to the extra dimension.

$$\beta_h^{ijk} = \gamma_l^i h^{ljk} + \gamma_l^j h^{ilk} + \gamma_l^k h^{ijl} - 2\gamma_{1l}^i Y^{ljk} - 2\gamma_{1l}^j Y^{ilk} - 2\gamma_{1l}^k Y^{ijl}\,, \tag{6.8}$$

$$(\beta_{m^2})_j^i = \left[2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2 \frac{\partial}{\partial g^2} + \tilde{Y}_{lmn}\frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}}\right]\gamma_j^i\,, \tag{6.9}$$

where $\beta_g = \Lambda\frac{dg}{d\Lambda}$ and etc., γ_j^i is the anomalous dimension given by

$$\gamma_j^i = \frac{1}{16\pi^2}\left[\frac{1}{2}\sum_{m,n} Y^{imn}Y_{jmn} - 2\delta_j^i g^2 C_2(R_i)\right], \tag{6.10}$$

$$\mathcal{O} = Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial Y^{lmn}}\,, \tag{6.11}$$

$$\tilde{Y}^{lmn} = (m^2)_k^l Y^{kmn} + (m^2)_k^m Y^{lkn} + (m^2)_k^n Y^{lmk}\,, \tag{6.12}$$

$\gamma_{1j}^i = \mathcal{O}\gamma_j^i$ and $Y_{lmn} = (Y^{lmn})^*$. Here $l(R_i)$ denotes the index of the representation R_i, and $C_2(G)$ and $C_2(R_i)$ are the quadratic Casimirs of the adjoint representation and the representation R_i of the gauge group G, respectively.

Now let us take the case with only gauge fields in the bulk for the running of couplings. Then, in the flavor bases where gaugino couplings are diagonal, bulk gauge corrections give rise to only the diagonal elements of anomalous dimensions. Above the GUT scale, we should take into account the KK modes of bulk gauge fields for the case without the GUT breaking. On the other hand, below the GUT scale down to the compactification scale, we have to include the GUT breaking effect to the KK modes of bulk gauge fields. Taking into account additional KK contributions to the 4D beta functions for soft parameters at each KK threshold, we find the following approximate one-loop renormalization group equations (RGEs)[6] for $\Lambda > M_{\rm GUT}$ and $M_c < \Lambda < M_{\rm GUT}$:

$$16\pi^2\beta_g = -2C_2(G)\left(\frac{\Lambda}{M_c}\right)g^3\,, \tag{6.13}$$

$$16\pi^2\beta_M = -4C_2(G)\left(\frac{\Lambda}{M_c}\right)g^2 M\,, \tag{6.14}$$

$$16\pi^2\beta_Y^{ijk} = \begin{cases} -2(C_2(R_i)+C_2(R_j)+C_2(R_k))\left(\frac{2\Lambda}{M_c}\right)g^2 Y^{ijk}\,, \\ -2\Big[C_2(R_i)+C_2(R_j)+C_2(R_k) \\ +(C_2^X(R_i)+C_2^X(R_j)+C_2^X(R_k))(f(\Lambda)-1)\Big]\left(\frac{2\Lambda}{M_c}\right)g^2 Y^{ijk}\,, \end{cases} \tag{6.15}$$

$$16\pi^2\beta_\mu^{ij} = \begin{cases} -2(C_2(R_i)+C_2(R_j))\left(\frac{2\Lambda}{M_c}\right)g^2\mu^{ij}\,, \\ -2\Big[C_2(R_i)+C_2(R_j) \\ +(C_2^X(R_i)+C_2^X(R_j))(f(\Lambda)-1)\Big]\left(\frac{2\Lambda}{M_c}\right)g^2\mu^{ij}\,, \end{cases} \tag{6.16}$$

$$16\pi^2\beta_B^{ij} = \begin{cases} 2(C_2(R_i)+C_2(R_j))\left(\frac{2\Lambda}{M_c}\right)g^2(2M\mu^{ij}-B^{ij})\,, \\ 2\Big[C_2(R_i)+C_2(R_j) \\ +(C_2^X(R_i)+C_2^X(R_j))(f(\Lambda)-1)\Big]\left(\frac{2\Lambda}{M_c}\right)g^2(2M\mu^{ij}-B^{ij})\,, \end{cases} \tag{6.17}$$

[6] Note that the beta functions for the gauge coupling and the gaugino mass are the half those in ref. [12]. They disregarded the reduction of the number of KK modes on S^1/Z_2.

$$16\pi^2\beta_h^{ijk} = \begin{cases} 2(C_2(R_i)+C_2(R_j)+C_2(R_k))\left(\frac{2\Lambda}{M_c}\right)g^2(2MY^{ijk}-h^{ijk})\,, \\ 2\Big[C_2(R_i)+C_2(R_j)+C_2(R_k) \\ +(C_2^X(R_i)+C_2^X(R_j)+C_2^X(R_k))(f(\Lambda)-1)\Big]\left(\frac{2\Lambda}{M_c}\right)g^2(2MY^{ijk}-h^{ijk}), \end{cases} \tag{6.18}$$

$$16\pi^2(\beta_{m^2})^i_j = \begin{cases} -8\delta^i_j C_2(R_i)\left(\frac{2\Lambda}{M_c}\right)g^2|M|^2\,, \\ -8\delta^i_j\Big[C_2(R_i)+C_2^X(R_i)(f(\Lambda)-1)\Big]\left(\frac{2\Lambda}{M_c}\right)g^2|M|^2 \end{cases} \tag{6.19}$$

where $C_2^X(R_i)$ is the broken part of the quadratic Casimir of the representation R_i. Note that $C_2^X(R_i)$ is different for the different representations of the unbroken gauge group in the same GUT multiplet. Therefore, for the nonzero factor $(f(\Lambda)-1)$, the runnings of soft mass parameters for the same GUT multiplet become non-universal between the GUT scale and the compactification scale.

Then, we find that the RGEs for M, Y^{ijk} and μ^{ij} for $M_c<\Lambda<M_{\rm GUT}$ are solved by

$$M(M_c) = \left(\frac{g(M_c)}{g(M_{\rm GUT})}\right)^2 M(M_{\rm GUT})\,, \tag{6.20}$$

$$Y^{ijk}(M_c) = \left(\frac{g(M_c)}{g(M_{\rm GUT})}\right)^{2\eta^{ijk}}\left(\frac{g(M_{\rm GUT})}{g(M_c)}\right)^{2t_1\eta_X^{ijk}} Y^{ijk}(M_{\rm GUT})\,, \tag{6.21}$$

$$\mu^{ij}(M_c) = \left(\frac{g(M_c)}{g(M_{\rm GUT})}\right)^{2\eta^{ij}}\left(\frac{g(M_{\rm GUT})}{g(M_c)}\right)^{2t_1\eta_X^{ij}} \mu^{ij}(M_{\rm GUT})\,, \tag{6.22}$$

where

$$\eta^{ijk} = \frac{C_2(R_i)+C_2(R_j)+C_2(R_k)}{C_2(G)}\,, \tag{6.23}$$

$$\eta^{ij} = \frac{C_2(R_i)+C_2(R_j)}{C_2(G)}\,, \tag{6.24}$$

$$t_1 = \left[\ln\frac{g(M_c)}{g(M_{\rm GUT})}\right]^{-1}\int_{g(M_c)}^{g(M_{\rm GUT})}(f(\Lambda)-1)\frac{dg}{g}\,, \tag{6.25}$$

and η_X^{ijk}, η_X^{ij} are the broken part of η^{ijk} and η^{ij}, respectively. For instance, in the SU(5) case, $\eta^{ijk}=48/25(42/25)$ for the up(down) type Yukawa coupling and $\eta^{ij}=24/25$. Note also that $\eta_X^{ijk}=1(4/5), 6/5$ for the up(down) type quark Yukawa coupling and the lepton Yukawa coupling in order while $\eta_X^{ij}=3/5(2/5)$ for the Higgs doublet(triplet). So, using the values, $\alpha(M_c)\simeq\frac{1}{23}$ for $M_c\simeq 10^{15}\,{\rm GeV}$, $g(M_c)/g(M_{\rm GUT})\simeq 2.8$ and $t_1=0.554$, the up type quark Yukawa coupling at M_c is lowered by 0.692 than the case without GUT breaking and the down type quark and lepton Yukawa couplings at M_c are different with $Y_d:Y_l=0.404:0.257$. The difference between μ parameters for the Higgs doublet and triplet at M_c comes with $\mu_{H_u}(=\mu_{H_d}):\mu_{H_C}(=\mu_{\bar H_C})=0.507:0.692$. Therefore, the GUT breaking effect only gives rise to the $O(1)$ difference between Yukawa couplings (or μ terms) in the same GUT multiplet at the compactification scale. However, compared to the case that the GUT scale is the same as the compactification scale [12], we find that the overall magnitude of Yukawa coupling and the μ term becomes even larger at M_c due to the difference between the GUT scale and the compactification scale.

On the other hand, the ratios for the SSB parameters B^{ij}, h^{ijk} and $(m^2)^i_j$ to the gaugino mass have the infrared fixed points shifted due to the GUT breaking effect compared to the case without the GUT breaking [12]. We also obtain the explicit solutions for those SSB parameters as

$$\frac{B^{ij}}{M\mu^{ij}}(M_c) = -2(\eta^{ij} - t_2\eta_X^{ij}) + \left(\frac{g(M_{\rm GUT})}{g(M_c)}\right)^2 \left(\frac{B^{ij}}{M\mu^{ij}}(M_{\rm GUT}) + 2\eta^{ij}\right), \tag{6.26}$$

$$\frac{h^{ijk}}{MY^{ijk}}(M_c) = -2(\eta^{ijk} - t_2\eta_X^{ijk}) + \left(\frac{g(M_{\rm GUT})}{g(M_c)}\right)^2 \left(\frac{h^{ijk}}{MY^{ijk}}(M_{\rm GUT}) + 2\eta^{ijk}\right), \tag{6.27}$$

$$\begin{aligned}\frac{(m^2)^i_j}{|M|^2}(M_c) &= \frac{2(C_2(R_i) - t_3 C_2^X(R_i))}{C_2(G)}\delta^i_j + \\ &+\left(\frac{g(M_{\rm GUT})}{g(M_c)}\right)^4 \left(\frac{(m^2)^i_j}{|M|^2}(M_{\rm GUT}) - \frac{2C_2(R_i)}{C_2(G)}\delta^i_j\right)\end{aligned} \tag{6.28}$$

where

$$t_2 = \frac{2}{g^2(M_c)}\int_{g(M_c)}^{g(M_{\rm GUT})} g^2(f(\Lambda)-1)\frac{dg}{g}\,, \tag{6.29}$$

$$t_3 = \frac{4}{g^4(M_c)}\int_{g(M_c)}^{g(M_{\rm GUT})} g^4(f(\Lambda)-1)\frac{dg}{g}\,. \tag{6.30}$$

Therefore, the diagonal elements of the soft scalar masses become dominant and degenerate. Moreover, inserting the following approximate value for the ratio of the gauge couplings into eq. (6.28) for $G = \mathrm{SU}(5)$

$$\frac{g(M_c)}{g(M_{\rm GUT})} \simeq \left[\frac{C_2(G)\alpha(M_c)}{\pi}\right]^{1/2}\left(\frac{M_{\rm GUT}}{M_c}\right)^{1/2} \simeq 2.63\,, \tag{6.31}$$

where we used $\alpha(M_c) \simeq \frac{1}{23}$ for $M_c \simeq 10^{15}$ GeV and $M_{\rm GUT}/M_c = 10^2$, the $O(1)$ off-diagonal components of $(m^2)^i_j/|M|^2$ initially given at $M_{\rm GUT}$ can be suppressed to be $O(10^{-2})$ as shown in the second term of eq. (6.28). So, for the contribution of the soft masses to $(\delta_{ii})_{LL,RR}$, we can satisfy the stringent bounds from the $K_L - K_S$ mass difference Δm_K and the decay $\mu \to e\gamma$ [2]. Likewise, from eq. (6.27), when the size of h^{ijk} is assumed to be of order MY^{ijk} at $M_{\rm GUT}$, we also obtain the non-aligned components of the trilinear couplings as follows

$$\left|\frac{h^{ijk}}{M}(M_c) + 2(\eta^{ijk} - t_2\eta_X^{ijk})Y^{ijk}(M_c)\right| \simeq \left(\frac{g(M_{\rm GUT})}{g(M_c)}\right)^2 O(Y^{ijk}(M_c))\,. \tag{6.32}$$

Thus, in the SU(5) case, we find that the non-aligned components of the trilinear couplings contribute to $(\delta_{ij})_{LR}$: $(\delta^{l(d)}_{ij})_{LR} \simeq \xi\langle H_d\rangle MY^{l(d)}_{ij}/m^2_{\tilde{l}(\tilde{q})}$ for the sleptons(down squarks) and $(\delta^u_{ij})_{LR} \simeq \xi\langle H_u\rangle MY^u_{ij}/m^2_{\tilde{q}}$ for the up squarks where $\xi \equiv (g(M_{\rm GUT})/g(M_c))^2$. Therefore, in the basis that the up quark mass matrix is diagonal [3], we obtain the LR mass terms contributing to ϵ'/ϵ, $b \to s\gamma$ and $\mu \to e\gamma$ decay processes in order as $\mathrm{Im}(\delta^d_{12})_{LR} \simeq \xi m_s V_{us} M/m^2_{\tilde{q}} \sim 2.9\times 10^{-6}$, $(\delta^d_{23})_{LR} \simeq \xi m_b V_{cb} M/m^2_{\tilde{q}} \sim 2.9\times 10^{-6}$ and $(\delta^l_{12})_{LR} \simeq \xi m_\mu V_{\nu_e\mu} \times M/m^2_{\tilde{l}} \sim 3.0\times 10^{-5} V_{\nu_e\mu}$ where V_{us}, V_{cb} and $V_{\nu_e\mu}$ denote quark and lepton mixings. Henceforth we take $m_{\tilde{q}} \sim m_{\tilde{l}} \sim 500$GeV and assume that the photino(gluino) mass is the same as the slepton(squark) mass. Therefore, we can satisfy or saturate the experimental limits on the LR mass terms such as $\mathrm{Im}(\delta^d_{12})_{LR} < 2.1 \times 10^{-5}$, $(\delta^d_{23})_{LR} < 1.6 \times 10^{-2}$, and $(\delta^l_{12})_{LR} < 4.3 \times 10^{-6}$.

The diagonal terms of trilinear couplings with the nonzero phases also contribute to $\mathrm{Im}(\delta_{ii})_{LR}$ [2]. The most stringent contraints on those come from the electric dipole moments of the neutron and the electron [2]: $\mathrm{Im}(\delta^l_{11})_{LR} < 9.3 \times 10^{-6}$ and $\mathrm{Im}(\delta^d_{11})_{LR} < 3.0 \times 10^{-6}$. Since the phases of h^{ijk}/MY^{ijk} given at $M_{\rm GUT}$ are suppressed to be of order 10^{-2} due to the power running from eq. (6.27), we obtain the sufficient suppression of LR mass terms to satisfy the EDM constraints as $\mathrm{Im}(\delta^l_{11})_{LR} \sim \xi M m_e/m^2_{\tilde{l}} \sim 1.5 \times 10^{-7}$ and $\mathrm{Im}(\delta^d_{11})_{LR} \sim \xi M m_d/m^2_{\tilde{q}} \sim 4.4 \times 10^{-7}$. Likewise, the phases of B terms are sufficiently suppressed at M_c. On the other hand, in the model with one bulk Higgs multiplet, which is needed for a successful unification of gauge couplings, the large difference of scales, $M_{\rm GUT}$ and M_c, is compatible with the successful unification of gauge couplings and the proton longevity. However, the flavor violation due to power-law Yukawa interaction could make the soft parameters non-universal. In the next section, let us deal with the power-law Yukawa terms giving rise to the flavor violation again.

For Λ close to M_c, the logarithmic corrections will not be negligible, which could destroy the universality of the SSB terms. However, in the concrete example of the SU(5) GUT, it has been shown that the logarithmic corrections coming from the Yukawa interactions are small enough to avoid the harmful flavor or CP violating processes [12].

For concreteness, let us consider the infrared fixed points of the soft mass parameters in the SU(5) model. The SSB lagrangian except for the gauginos is given by

$$\begin{aligned}
-\mathcal{L}_{SSB} = & \; m^2_H H^\dagger H + m^2_{\bar{H}} \bar{H}^\dagger \bar{H} + m^2_\Sigma \mathrm{tr}(\Sigma^\dagger \Sigma) + \\
& + \sum_{i,j}^{3} \Big[(m^2_{\Psi(\bar{5})})^{ij} \Psi_i(\bar{5})^\dagger \Psi_j(\bar{5}) + (m^2_{\Psi(10)})^{ij} \mathrm{tr}(\Psi_i(10)^\dagger \Psi_j(10)) \Big] + \\
& + \Big\{ B_\Sigma \mathrm{tr}\Sigma^2 + \frac{h_\lambda}{3} \mathrm{tr}\Sigma^3 + h_f \bar{H}\Sigma H + B_H \bar{H} H + \\
& \quad + \frac{h^{ij}_U}{4} \Psi_i(10)\Psi_j(10) H + \sqrt{2} h^{ij}_D \Psi_i(\bar{5}) \Psi_j(10) \bar{H} + \mathrm{h.c.} \Big\}
\end{aligned} \tag{6.33}$$

where it is understood that all fields are scalar components of the corresponding superfields. Then, due to the GUT breaking effect, the soft mass parameters for the same GUT multiplet have different infrared fixed points. Using the general formulae, eqs. (6.26)-(6.28), and the numerical values $t_2 = 0.479$ and $t_3 = 0.544$, the SSB parameters approach the following values in the limit of the energy scale going down to the compactification scale:

$$B_{H_u}, B_{H_d} \to -\frac{48}{25} M \mu_{H_u} \left(1 - \frac{5}{8} t_2\right) = -1.35 M \mu_{H_u}\,, \tag{6.34}$$

$$B_{H_C}, B_{H_{\bar{C}}} \to -\frac{48}{25} M \mu_{H_C} \left(1 - \frac{5}{12} t_2\right) = -1.54 M \mu_{H_C}\,, \tag{6.35}$$

$$h_U \to -\frac{96}{25} M Y_U \left(1 - \frac{25}{48} t_2\right) = -2.88 M Y_U\,, \tag{6.36}$$

$$h_d \to -\frac{84}{25} M Y_d \left(1 - \frac{10}{21} t_2\right) = -2.59 M Y_d\,, \tag{6.37}$$

$$h_l \to -\frac{84}{25} M Y_l \left(1 - \frac{5}{7} t_2\right) = -2.21 M Y_l\,, \tag{6.38}$$

$$m^2_{\tilde{q}} \to \frac{36}{25} |M|^2 \left(1 - \frac{5}{12} t_3\right) = 1.11 |M|^2\,, \tag{6.39}$$

$$m^2_{\tilde{u}^c} \to \frac{36}{25} |M|^2 \left(1 - \frac{5}{9} t_3\right) = 1.01 |M|^2\,, \tag{6.40}$$

$$m^2_{\tilde{e}^c} \to \frac{36}{25} |M|^2 \left(1 - \frac{5}{6} t_3\right) = 0.788 |M|^2\,, \tag{6.41}$$

$$m^2_{\tilde{d}^c}, m^2_{H_C}, m^2_{H_{\bar{C}}} \to \frac{24}{25} |M|^2 \left(1 - \frac{5}{12} t_3\right) = 0.742 |M|^2\,, \tag{6.42}$$

$$m^2_{\tilde{l}}, m^2_{H_u}, m^2_{H_d} \to \frac{24}{25} |M|^2 \left(1 - \frac{5}{8} t_3\right) = 0.634 |M|^2\,, \tag{6.43}$$

where we considered soft parameters in terms of components of the GUT multiplet under the SM group representations. Note that at the GUT scale, $\mu_{H_u} = \mu_{H_C} = \mu_H$, $B_{H_u} = B_{H_C} = B_H$, $B_{H_d} = B_{H_{\bar{C}}} = B_{\bar{H}}$, $h_d = h_l = h_D$, $Y_d = Y_l = Y_D$, $m^2_{\tilde{q}} = m^2_{\tilde{u}^c} = m^2_{\tilde{e}^c} = m^2_{\Psi(10)}$, $m^2_{\tilde{d}^c} = m^2_{\tilde{l}} = m^2_{\Psi(\bar{5})}$, $m^2_{H_u} = m^2_{H_C} = m^2_H$, and $m^2_{H_d} = m^2_{H_{\bar{C}}} = m^2_{\bar{H}}$.

7. Flavor universality with bulk Higgs multiplet(s)

In the section 4, we have shown that letting Higgs fields propagate in the bulk is needed to get a more successful unification of gauge couplings and it is also possible to have a large separation between the GUT scale and the compactification scale. However, introduction of bulk Higgs fields would lead to a power-law contribution of additional KK modes to the RGEs, which does not respect the flavor universality. Moreover, as mentioned in the previous section, there is no gauge correction to the anomalous dimension of the zero mode of a bulk Higgs field due to the $\mathcal{N} = 2$ non-renormalization theorem. Therefore, in this section, we will present the RGEs for soft parameters in the case with bulk Higgs field(s) and show how infrared fixed points of soft parameters can be maintained.

In the case with bulk Higgs field(s),[7] we use the 4D results to obtain the anomalous dimensions for the brane matters **10**, $\bar{\mathbf{5}}$ and bulk Higgs field(s) [19]. Here we consider the case (1) with the down type Higgs in the bulk and the up type Higgs on the brane and the case (2) with both two Higgs fields in the bulk. Then, when we neglect the $O(1)$ GUT breaking effect for the time being, we can get one-loop anomalous dimensions above the compactification scale with the upper and lower ones in the braces for the cases (1) and

[7]Note that we have to put a SUSY breaking mass for bulk Higgs scalar(s) at $y = \pi R$ for softness as for bulk gauginos. Then, the brane SUSY breaking parameter for bulk Higgs scalar(s) has a mass dimension of one. Of course, a brane Higgs has a soft mass at $y = 0$.

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(2), respectively,

$$16\pi^2\gamma_{10} = \begin{cases} (-\frac{36}{5}g^2 + 2Y_D Y_D^\dagger)\left(\frac{2\Lambda}{M_c}\right) + 3Y_U Y_U^\dagger \\ (-\frac{36}{5}g^2 + 3Y_U Y_U^\dagger + 2Y_D Y_D^\dagger)\left(\frac{2\Lambda}{M_c}\right), \end{cases} \tag{7.1}$$

$$16\pi^2\gamma_{\bar{5}} = \left(-\frac{24}{5}g^2 + 4Y_D Y_D^\dagger\right)\left(\frac{2\Lambda}{M_c}\right), \tag{7.2}$$

$$16\pi^2\gamma_H = \begin{cases} -\frac{24}{5}g^2\left(\frac{2\Lambda}{M_c}\right) + 3\mathrm{Tr}Y_U Y_U^\dagger \\ 3\mathrm{Tr}Y_U Y_U^\dagger, \end{cases} \tag{7.3}$$

$$16\pi^2\gamma_{\bar{H}} = 4\mathrm{Tr}Y_D Y_D^\dagger. \tag{7.4}$$

Note that there are additional power-law corrections to the anomalous dimensions of brane matters **10**, $\bar{\mathbf{5}}$ due to the KK modes of bulk Higgs multiplet(s). On the other hand, the zero mode of a bulk Higgs field only obtains a logarithmic anomalous dimension from the brane matters via the Yukawa couplings as shown in the lower one of eq. (7.3) and eq. (7.4).

Consequently, using the above anomalous dimensions and the 4D beta functions and keeping only the dominant part of one-loop beta functions, we obtain those for the rigid supersymmetry parameters as

$$16\pi^2\beta_g = \begin{cases} -9\left(\frac{\Lambda}{M_c}\right)g^3, \\ -8\left(\frac{\Lambda}{M_c}\right)g^3, \end{cases} \tag{7.5}$$

$$16\pi^2\beta_M = \begin{cases} -18\left(\frac{\Lambda}{M_c}\right)g^2 M, \\ -16\left(\frac{\Lambda}{M_c}\right)g^2 M, \end{cases} \tag{7.6}$$

$$16\pi^2\beta_{Y_U} = \begin{cases} \left(-\frac{96}{5}g^2 + 4Y_D Y_D^\dagger\right) Y_U\left(\frac{2\Lambda}{M_c}\right), \\ \left(-\frac{72}{5}g^2 + 6Y_U Y_U^\dagger + 4Y_D Y_D^\dagger\right) Y_U\left(\frac{2\Lambda}{M_c}\right), \end{cases} \tag{7.7}$$

$$16\pi^2\beta_{Y_D} = \begin{cases} (-12g^2 + 6Y_D Y_D^\dagger)Y_D\left(\frac{2\Lambda}{M_c}\right), \\ (-12g^2 + 3\mathrm{Tr}Y_U Y_U^\dagger + 6Y_D Y_D^\dagger)Y_D\left(\frac{2\Lambda}{M_c}\right), \end{cases} \tag{7.8}$$

$$16\pi^2\beta_\mu = \begin{cases} -\frac{24}{5}g^2\left(\frac{2\Lambda}{M_c}\right)\mu, \\ (3\mathrm{Tr}Y_U Y_U^\dagger + 4\mathrm{Tr}Y_D Y_D^\dagger)\mu, \end{cases} \tag{7.9}$$

and those for the soft supersymmetry breaking parameters as

$$16\pi^2\beta_B = \begin{cases} \frac{24}{5}g^2\left(\frac{2\Lambda}{M_c}\right)(2\mu M - B), \\ (3\mathrm{Tr}Y_U Y_U^\dagger + 4\mathrm{Tr}Y_D Y_D^\dagger)B + (6\mathrm{Tr}h_U Y_U^\dagger + 8\mathrm{Tr}h_D Y_D^\dagger)\mu, \end{cases} \tag{7.10}$$

$$
16\pi^2\beta_{h_U} = \begin{cases} \left[\frac{96}{5}g^2(2MY_U - h_U) + 4Y_DY_D^\dagger h_U + 8(\mathrm{Tr}h_DY_D^\dagger)Y_U\right]\left(\frac{2\Lambda}{M_c}\right), \\ \left[\frac{72}{5}g^2(2MY_U - h_U) + (6Y_UY_U^\dagger + 4Y_DY_D^\dagger)h_U \right. \\ \qquad \left. +(12\mathrm{Tr}h_UY_U^\dagger + 8\mathrm{Tr}h_DY_D^\dagger)Y_U\right]\left(\frac{2\Lambda}{M_c}\right)\mu\,, \end{cases} \tag{7.11}
$$

$$
16\pi^2\beta_{h_D} = \begin{cases} \left[12g^2(2MY_D - h_D) + 6Y_DY_D^\dagger h_D + 12(\mathrm{Tr}h_DY_D^\dagger)Y_D\right]\left(\frac{2\Lambda}{M_c}\right), \\ \left[12g^2(2MY_D - h_D) + (3Y_UY_U^\dagger + 6Y_DY_D^\dagger)h_D \right. \\ \qquad \left. +(6\mathrm{Tr}h_UY_U^\dagger + 12\mathrm{Tr}h_DY_D^\dagger)Y_D\right]\left(\frac{2\Lambda}{M_c}\right)\mu\,, \end{cases} \tag{7.12}
$$

$$
16\pi^2\beta_{m^2_{10}} = \begin{cases} \left[-\frac{144}{5}g^2|M|^2 + 4h_Dh_D^\dagger + 4(m^2_{10} + m^2_{\bar{5}} + m^2_{H_d})Y_DY_D^\dagger\right]\left(\frac{2\Lambda}{M_c}\right), \\ \left[-\frac{144}{5}g^2|M|^2 + 6h_Uh_U^\dagger + 4h_Dh_D^\dagger + 6(2m^2_{10} + m^2_{H_u})Y_UY_U^\dagger \right. \\ \qquad \left. +4(m^2_{10} + m^2_{\bar{5}} + m^2_{H_d})Y_DY_D^\dagger\right]\left(\frac{2\Lambda}{M_c}\right), \end{cases} \tag{7.13}
$$

$$
16\pi^2\beta_{m^2_{\bar{5}}} = \left[-\frac{96}{5}g^2|M|^2 + 8h_Dh_D^\dagger + 8(m^2_{10} + m^2_{\bar{5}} + m^2_{H_u})Y_DY_D^\dagger\right]\left(\frac{2\Lambda}{M_c}\right), \tag{7.14}
$$

$$
16\pi^2\beta_{m^2_{H_u}} = \begin{cases} -\frac{96}{5}g^2|M|^2\left(\frac{2\Lambda}{M_c}\right), \\ 6\mathrm{Tr}h_Uh_U^\dagger + 6(2m^2_{10} + m^2_{H_u})\mathrm{Tr}Y_UY_U^\dagger\,, \end{cases} \tag{7.15}
$$

$$
16\pi^2\beta_{m^2_{H_d}} = 8\mathrm{Tr}h_Dh_D^\dagger + 8(m^2_{10} + m^2_{\bar{5}} + m^2_{H_d})\mathrm{Tr}Y_DY_D^\dagger\,. \tag{7.16}
$$

Note that only for the case (1) with the down type Higgs field in the bulk, we can retain the power law contribution to the beta function of the B term or the μ term, so that there does exist an IR fixed point for the B term which lead to a suppressed phase of B. More importantly, for the case (1), additional power terms coming only from the down type Yukawa coupling can be negligible for universality of soft masses at the compactification scale. On the other hand, for the case (2) with both Higgs fields in the bulk, the power running due to the up type Yukawa coupling could be of the same order of magnitude as that due to the gauge interaction, for instance, near the compactification scale. Therefore, we cannot ignore the potential non-universality due to the up type Yukawa coupling in the case (2). Therefore, henceforth let us concentrate on the case (1), i.e. the upper one of each pair of beta functions.

Even with one bulk Higgs, we find that the gauge coupling has the power running with asymptotic freedom and a standard relation between gauge coupling and gaugino mass holds. Moreover, since there is one Higgs field at the brane, μ, B terms and the soft mass of the up type Higgs scalar have power contributions only from KK modes of gauge fields. On the other hand, Yukawa couplings and other soft mass parameters have the power contributions due to the KK modes of the bulk Higgs field via the down type Yukawa interactions, which are flavor dependent. However, we can show that the power-like disorder arising from Yukawa couplings can be negligible due to the smallness of the down type Yukawa couplings.

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To begin with, let us consider the disorder in the Yukawa couplings due to the KK modes of the down type bulk Higgs. For simplicity, let us neglect the $O(1)$ GUT breaking effect. By using eq. (7.5), we can rewrite eqs. (7.7) and (7.8) as

$$16\pi^2\Lambda\frac{d}{d\Lambda}\tilde{Y}_U = 4Y_DY_D^\dagger\tilde{Y}_U\left(\frac{2\Lambda}{M_c}\right), \tag{7.17}$$

$$16\pi^2\Lambda\frac{d}{d\Lambda}\tilde{Y}_D = 6Y_DY_D^\dagger\tilde{Y}_D\left(\frac{2\Lambda}{M_c}\right), \tag{7.18}$$

where

$$\tilde{Y}_U \equiv \frac{Y_U}{g^{2\eta_U}}, \qquad \tilde{Y}_D \equiv \frac{Y_D}{g^{2\eta_D}} \tag{7.19}$$

with $\eta_U = 32/15$ and $\eta_D = 4/3$. Then, inserting the solutions for Yukawa couplings without Yukawa power corrections such as

$$\tilde{Y}_U(\Lambda) = \tilde{Y}_U(M_c), \qquad \tilde{Y}_D(\Lambda) = \tilde{Y}_D(M_c), \tag{7.20}$$

into the right-hand sides of eqs. (7.17) and (7.18), and using $Y_D/Y_U \simeq 1/60$ with $Y_U \simeq 1$ at M_c and $\alpha(M_c) \simeq \frac{1}{23}$ for $M_c \simeq 10^{15}$ GeV, we can estimate the power-like disorder in the Yukawa couplings as

$$\tilde{Y}_U^{-1}(M_c)\tilde{Y}_U(\Lambda) - 1 \lesssim \frac{4g^{-2}(M_c)}{9(2\eta_D-1)}(Y_U^{-1}Y_DY_D^\dagger Y_U)(M_c) \sim 10^{-4}, \tag{7.21}$$

$$\tilde{Y}_D^{-1}(M_c)\tilde{Y}_D(\Lambda) - 1 \lesssim \frac{2g^{-2}(M_c)}{3(2\eta_D-1)}(Y_D^\dagger Y_D)(M_c) \sim 10^{-4}. \tag{7.22}$$

On the other hand, the power contribution to the μ term comes only from the gauge interaction as can be seen in the upper one of eq. (7.9). Thus, we get the similar running of the μ term with a different exponent of g compared to the case without bulk Higgs: $\eta^{ij} = 24/25 \to \eta_\mu = 2C_2(5)/9 = 8/15$ in eq. (6.22).

Before going into the evolution of the soft mass parameters of brane matters, let us consider the RG flow of the down type Higgs mass. The zero mode of the down type bulk Higgs only has the nonzero anomalous dimension via the Yukawa coupling with the brane matters as shown in eq. (7.16). Now we show how the running of the down type Higgs mass can be neglected above the compactification scale by taking its ratio to the Yukawa couplings. Here we do not include the GUT breaking effect to show the gross behavior. Using eq. (7.16) and the upper one of eq. (7.8) and recalling that the dominant running masses of the brane scalars are of order the gaugino mass, we obtain the following approximate running equation for $\tilde{m}^2_{H_d} = m^2_{H_d}/(\mathrm{Tr}Y_DY_D^\dagger)$

$$\begin{aligned}16\pi^2\Lambda\frac{d}{d\Lambda}\tilde{m}^2_{H_d} \simeq\ & 8(1+a+b)|M|^2 + 8m^2_{H_d} + 24g^2\tilde{m}^2_{H_d}\left(\frac{2\Lambda}{M_c}\right) - \\ & -12\frac{\mathrm{Tr}(Y_DY_D^\dagger Y_DY_D^\dagger)}{\mathrm{Tr}(Y_DY_D^\dagger)}\tilde{m}^2_{H_d}\left(\frac{2\Lambda}{M_c}\right)\end{aligned} \tag{7.23}$$

where a, b are $O(1)$ dimensionless quantities. Thus, neglecting the first two log terms in the above equation and using eq. (7.5), we can find the approximate solution for the above equation as

$$\ln\left(\frac{\tilde{m}_{H_d}^2 g^{\frac{16}{3}}(\Lambda)}{\tilde{m}_{H_d}^2 g^{\frac{16}{3}}(M_c)}\right) \lesssim \frac{4g^{-2}(M_c)}{3(2\eta_D-1)} \frac{\mathrm{Tr}(Y_D Y_D^\dagger Y_D Y_D^\dagger)}{\mathrm{Tr}(Y_D Y_D^\dagger)} \sim 10^{-4} \tag{7.24}$$

where we used the approximate solution for Y_D in eq. (7.20). As a result, we can find the RG evolution of the down type Higgs mass negligible compared to other parameters with power running: $\tilde{m}_{H_d}^2 g^{\frac{16}{3}}$ is almost constant, so is $m_{H_d}^2 \propto (\mathrm{Tr} Y_D Y_D^\dagger) \cdot g^{-\frac{16}{3}} \propto g^{4\eta_D - \frac{16}{3}} = g^0$.

By restoring the GUT breaking effect between the GUT scale and M_c in the same way as in the previous section, we can find the following flavor conserving behavior of soft mass parameters in the infra-red limit and their deviations at M_c as follows:

$$\frac{B}{M\mu}(M_c) = -2\left(\eta_\mu - t_2\eta_\mu^X\right) + \left(\frac{g(M_{\mathrm{GUT}})}{g(M_c)}\right)^2 \left(\frac{B}{M\mu}(M_{\mathrm{GUT}}) + 2\eta_\mu\right), \tag{7.25}$$

$$\begin{aligned}\frac{h_U}{MY_U}(M_c) &= -2(\eta_U - t_2\eta_U^X) + \\ &+\left(\frac{g(M_{\mathrm{GUT}})}{g(M_c)}\right)^2 \left(\frac{h_U}{MY_U}(M_{\mathrm{GUT}}) + 2\eta_U\right) + \delta_{h_U}(M_c)\,,\end{aligned} \tag{7.26}$$

$$\begin{aligned}\frac{h_D}{MY_D}(M_c) &= -2(\eta_D - t_2\eta_D^X) + \\ &+\left(\frac{g(M_{\mathrm{GUT}})}{g(M_c)}\right)^2 \left(\frac{h_D}{MY_D}(M_{\mathrm{GUT}}) + 2\eta_D\right) + \delta_{h_D}(M_c)\,,\end{aligned} \tag{7.27}$$

and

$$\begin{aligned}\frac{(m_{10}^2)^i_j}{|M|^2}(M_c) &= \frac{4}{9}(C_2(10) - t_3 C_2^X(10))\delta^i_j \\ &+\left(\frac{g(M_{\mathrm{GUT}})}{g(M_c)}\right)^4 \left(\frac{(m_{10}^2)^i_j}{|M|^2}(M_{\mathrm{GUT}}) - \frac{4}{9}C_2(10)\delta^i_j\right) + (\delta_{10})^i_j(M_c)\,,\end{aligned} \tag{7.28}$$

$$\begin{aligned}\frac{(m_{\bar{5}}^2)^i_j}{|M|^2}(M_c) &= \frac{4}{9}(C_2(\bar{5}) - t_3 C_2^X(\bar{5}))\delta^i_j + \\ &+\left(\frac{g(M_{\mathrm{GUT}})}{g(M_c)}\right)^4 \left(\frac{(m_{\bar{5}}^2)^i_j}{|M|^2}(M_{\mathrm{GUT}}) - \frac{4}{9}C_2(\bar{5})\delta^i_j\right) + (\delta_{\bar{5}})^i_j(M_c)\,,\end{aligned} \tag{7.29}$$

$$\begin{aligned}\frac{m_{H_u}^2}{|M|^2}(M_c) &= \frac{4}{9}(C_2(5) - t_3 C_2^X(5)) \\ &+\left(\frac{g(M_{\mathrm{GUT}})}{g(M_c)}\right)^4 \left(\frac{m_{H_u}^2}{|M|^2}(M_{\mathrm{GUT}}) - \frac{4}{9}C_2(5)\right),\end{aligned} \tag{7.30}$$

where

$$\begin{aligned}|\delta_{h_U}(M_c)| &\simeq \left|\frac{16}{9g^2(M_c)}\int_{g(M_c)}^{g(M_{\mathrm{GUT}})} \frac{g^2}{M}(\mathrm{Tr} h_D Y_D^\dagger)\frac{dg}{g^3}\right|, \\ &\lesssim \frac{8}{9}\frac{|\mathrm{Tr} Y_D Y_D^\dagger|}{g^2}(M_c) \sim 10^{-4}\,,\end{aligned} \tag{7.31}$$

$$
\begin{aligned}
|\delta_{h_D}(M_c)| &\simeq \left| \frac{8}{3g^2(M_c)} \int_{g(M_c)}^{g(M_{\rm GUT})} \frac{g^2}{M} ({\rm Tr} h_D Y_D^\dagger) \frac{dg}{g^3} \right| , \\
&\lesssim \frac{4}{3} \frac{|{\rm Tr} Y_D Y_D^\dagger|}{g^2} (M_c) \sim 10^{-4} ,
\end{aligned} \tag{7.32}
$$

and

$$
\begin{aligned}
&|(\delta_{10})^i_j(M_c)| = \frac{1}{2} |(\delta_{\bar{5}})^i_j(M_c)| \\
&\simeq \left| \frac{8}{9g^4(M_c)} \int_{g(M_c)}^{g(M_{\rm GUT})} \frac{g^4}{|M|^2} ((h_D h_D^\dagger)^i_j + (m^2_{10} + m^2_{\bar{5}} + m^2_{H_d})(Y_D Y_D^\dagger)^i_j) \frac{dg}{g^3} \right| \\
&\lesssim \frac{|(Y_D Y_D^\dagger)^i_j|}{g^2} (M_c) \sim 10^{-4} .
\end{aligned} \tag{7.33}
$$

Here we find the different measure of the non-universal running of soft parameters compared to the case without bulk Higgs: $\eta^X_{ij} = 1/3(2/9)$ for Higgs doublet(triplet), $\eta^X_U = 10/9$ and $\eta^X_D = 7/9(1/3)$ for down type quark(lepton) Yukawa coupling. Note that when we made approximations for the corrections due to the bulk Higgs, we used the approximate solutions for Y_U and Y_D given by eq. (7.20) and the fact that $h_U \sim MY_U$ and $h_D \sim MY_D$ at M_c. For instance, we inserted the dominant evolution of soft masses due to gauge interaction into the right-hand sides of eqs. (7.31)-(7.33). We also find that the dominant flavor violation due to the bulk Higgs comes from the KK modes near the compactification scale, which can be seen from the scale-independent bounds on the corrections as in eqs. (7.31)-(7.33). The reason is that as the energy scale increases, the Yukawa couplings powerly run into small values due to the dominant contribution from the KK modes of gauge fields, so the highest KK modes of the bulk Higgs are weakly coupled to the brane matters.

Consequently, we can find that independently of the precise separation between the GUT scale and the compactification scale, the additional corrections with flavor dependence to the soft masses of brane scalars are of order less than 10^{-4}, which is small enough to satisfy the experimental limits. When we assume that $h_U(h_D)$ is of order $MY_U(MY_D)$ at the GUT scale, we also find that the generated non-aligned components of trilinear couplings are of order less than 10^{-4} and 10^{-6} for h_U and h_D, respectively, which are also small enough for good agreement with experiments.

On the other hand, we still get the suppression of phases due to the power corrections even with the bulk Higgs. In the case (1) which we are interested in, the phases of the trilinear couplings and the B term are still suppressed to be of order 10^{-2} via the power running due to the bulk gauge multiplet. We also obtain the additional phases of trilinear couplings to be of order 10^{-4} due to the KK modes of the bulk Higgs multiplet. So, we end up with the sufficiently suppressed CP violation even in the case with one bulk Higgs.

8. Conclusion

We have shown that in 5D SUSY SU(5) GUT on S^1/Z_2, the difference between the GUT scale and the compactification scale gives the non-universal power running of soft masses such that their flavor dependent part can be negligible in the infrared limit. Firstly, for our purpose, we have taken the simple choice with only gauge fields in the bulk and matter and Higgs fields on the brane. We included the GUT breaking effect for the renormalization group evolution of soft masses and showed that it makes the difference of less than $O(1)$ in the fixed point values of soft mass parameters for the same GUT multiplet. Next, we found that with the addition of bulk Higgs field of down type, we not only explain the top-bottom mass hierarchy but also obtain a successful unification of gauge couplings due to the KK modes of the bulk Higgs multiplet. In either case without or with a bulk Higgs, the $O(1)$ CP phases of soft mass parameters given at $M_{\rm GUT}$ are sufficiently suppressed to be of order 10^{-2} to satisfy the stringent bounds from the EDMs.

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